UNCLASSIFIED

AD NUMBER

AD069089

NEW LIMITATION CHANGE

TO

Approved for public release, distribution unlimited

FROM

Distribution authorized to U.S. Gov't. agencies and their contractors; Administrative/Operational Use; MAY 1955. Other requests shall be referred to Office of Naval Research, Arlington, VA 22203.

AUTHORITY

ONR memo., 1 Aug 1967

THIS PAGE IS UNCLASSIFIED

69089

Armed Services Technical Information Agency

Reproduced by
DOCUMENT SERVICE CENTER
KNOTT BUILDING, DAYTON, 2, OHIO

NOTICE: WHEN GOVERNMENT OR OTHER DRAWINGS, SPECIFICATIONS OR OTHER DATA ARE USED FOR ANY PURPOSE OTHER THAN IN CONNECTION WITH A DEFINITELY RELATED GOVERNMENT PROCUREMENT OPERATION, THE U. S. GOVERNMENT THEREBY INCURS NO RESPONSIBILITY, NOR ANY OBLIGATION WHATSOEVER; AND THE FACT THAT THE GOVERNMENT MAY HAVE FORMULATED, FURNISHED, OR IN ANY WAY SUPPLIED THE SAID DRAWINGS, SPECIFICATIONS, OR OTHER DATA IS NOT TO BE REGARDED BY IMPLICATION OR OTHERWISE AS IN ANY MANNER LICENSING THE HOLDER OR ANY OTHER PERSON OR CORPORATION, OR CONVEYING ANY RIGHTS OR PERMISSION TO MANUFACTURE, USE OR SELL ANY PATENTED INVENTION THAT MAY IN ANY WAY BE RELATED THERETO.

UNCLASSIFIED

AD No. 64089
ASTIA FILE COPY

FC

Office of Naval Research

Contract N5ORI-76 • Task Order No.1 • NR-372-012

# IMPEDANCE OF THIN WIRE LOOP ANTENNA

By

James E. Storer

May 1, 1955

Technical Report No. 212

Cruft Laboratory
Harvard University
Cambridge, Massachusetts

TR212

Office of Naval Research

Contract N5ori-76

Task Order No. 1

NR-372-012

Technical Report

on

Impedance of Thin Wire Loop Antennas

James E. Storer

May 1, 1955

The research reported in this document was made possible through support extended Cruft Laboratory, Harvard University, jointly by the Navy Department (Office of Naval Research), the Signal Corps of the U. S. Army and the U. S. Air Force, under ONR Contract N5ori-76, T.O. 1.

Technical Report No. 212

Cruft Laboratory

Harvard University

Cambridge, Massachusetts

## Abstract

The Hallén integral equation for the current and impedance of a thin wire loop antenna is solved using a Fourier Series. Extensive tables of theoretical loop antenna impedances are presented which (for the one case tested) are in satisfactory agreement with experiment. Some graphical results are also given which facilitate the evaluation of the current distribution.

TR212

# Impedance of Thin Wire Loop Antennas

by

James E. Storer

Cruft Laboratory, Harvard University

## I

## Introduction

The thin wire loop is one of the first antennas to receive theoretical consideration, having been discussed by Pocklington [1] in 1897. Pocklington treated a closed loop excited by a plane wave; he obtained an exact solution for the current on the loop in the form of a Fourier series. More recently, Hallén [2] considered a driven loop and obtained a solution, again in the form of a Fourier series, for the current and the impedance. However, Hallén pointed out that the coefficients of this series contained a singularity which made the series only quasiconvergent and hence useful only for loops small in comparison to a wavelength. Moreover, the individual terms were complicated and their evaluation and a summation involved a somewhat difficult numerical task.

More recently, in an effort to obtain numerical results, other authors have dealt with the problem using approximation methods. Chang [3], for example, applied the Hallén-King-Middleton expansion; Schelkunoff [4] has used a guided-mode approximation; and the author (unpublished) has used a variational approach. All of these approximation methods have one feature in common; they yield results which are in good agreement qualitatively with experiment, but poor agreement quantitatively.* The reason for this can be explained by noting that all the approximation methods require some assumption as to the current distribution around the loop. The most common

------

*It is quite possible that all of these methods, particularly Chang's, could be made to yield better results by going to higher degrees of approximation; the resulting numerical labor, however, is likely to be prohibitive.

assumption made is that the current distribution approximates a sinusoidal distribution. As will be shown subsequently, the sinusoidal assumption is not satisfactory, particularly for the current near the driving point of the antenna.

In the present paper the rigorous Fourier series solution obtained by Hallén is reexamined, and modified so that the convergence difficulties encountered by Hallén are avoided. Extensive numerical results are presented in Appendix II for the impedances of loops for varying wire sizes and circumferences up to two and one-half wavelengths. Appendix III presents some curves which aid in the computation of field patterns and current distributions. For an antenna having a particular wire size, some experimentally measured impedances are presented which agree well with theory.

## II

## Fourier Series Solution for the Current Distribution

Integral equations for the current distribution on thin-wire antenna structures are readily obtained by expressing the electric field as a function of the current, through Helmholtz integrals, and then equating the total electric field to zero along the wire surface. Following this procedure, with harmonic time dependence of the form $e^{+j\omega t}$, and with coordinate system and dimensions as indicated in Fig. 1.1, the integral equation for the circular loop antenna

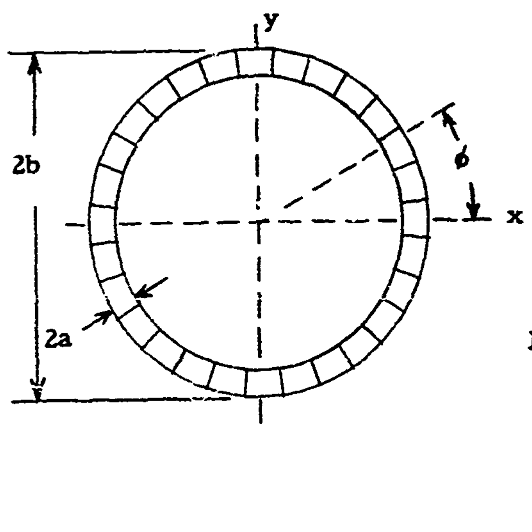



Figure 1.1

can be written as

$$V\delta(\phi) = \frac{j\zeta_0}{4\pi} \int_{-\pi}^{\pi} K(\phi-\phi')I(\phi')d\phi' \tag{1}$$

where $I(\phi)$ is the total current at $\phi$ on the loop; $V$ is the voltage of the slice generator exciting the loop at $\phi = 0$; $\delta(\phi)$ is the Dirac delta-function; and $k = \omega/c = 2\pi/\lambda$; $\zeta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = 120\pi$ ohms. The kernel of the integral equation, (1) is given explicitly by

$$K(\phi-\phi') = \left\{ kb\cos(\phi-\phi') + \frac{1}{kb}\frac{\partial^2}{\partial\phi^2} \right\} \frac{e^{-jkbR(\phi-\phi')}}{R(\phi-\phi')} \tag{2a}$$

$$R(\phi-\phi') = \left[4\sin^2\left(\frac{\phi-\phi'}{2}\right) + a^2/b^2\right]^{\frac{1}{2}} \tag{2b}$$

where a is the radius of the wire and b is the radius of the loop.

The thin-wire assumption, which provides the basis for obtaining this one-dimensional current equation, can be expressed explicitly as $a^2 \ll b^2$, $k^2a^2 \ll 1$. The resulting solution cannot be more accurate than the order of these approximations.

Since $\frac{1}{R(\phi-\phi')} e^{-jkbR(\phi-\phi')}$ is a bounded, periodic function of $\phi$, it can be expanded into a Fourier series, i.e.,

$$\frac{1}{R(\phi-\phi')} e^{-jkbR(\phi-\phi')} = \sum_{-\infty}^{\infty} K_n e^{-jn(\phi-\phi')} \tag{3}$$

$$K_n = K_{-n} = \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{e^{-jkbR(\phi)}}{R(\phi)} e^{-jn\phi} d\phi \tag{4}$$

Using (3) together with (2), it is seen that

$$K(\phi-\phi') = \sum_{-\infty}^{\infty} a_n e^{jn(\phi-\phi')}, \tag{5}$$

where

$$a_n = a_{-n} = kb\left\{\frac{K_{n+1} + K_{n-1}}{2}\right\} - \frac{n^2}{kb} K_n \qquad (6)$$

Inserting expression (5) into integral equation (1) yields

$$V\delta(\phi) = \frac{j\zeta_o}{4\pi} \sum_{-\infty}^{\infty} a_n \int_{-\pi}^{\pi} e^{jn(\phi-\phi')} I(\phi')d\phi' \qquad (7)$$

After expanding $I(\phi)$ into a Fourier series,

$$I(\phi) = \sum_{-\infty}^{\infty} I_n e^{jn\phi}; \quad I_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} I(\phi) e^{-jn\phi} d\phi \qquad (8)$$

it can be seen that (7) reduces to

$$V\delta(\phi) = \frac{j\zeta_o}{2} \sum_{-\infty}^{\infty} a_n I_n e^{jn\phi}$$

Hence

$$I_n = \frac{1}{j\pi\zeta_o a_n} \int_{-\pi}^{\pi} e^{-jn\phi} V\delta(\phi)d\phi = \frac{V}{j\pi\zeta_o a_n}$$

Inserting this result into equation (8) yields

$$I(\phi) = \frac{V}{j\pi\zeta_o} \sum_{-\infty}^{\infty} \frac{e^{jn\phi}}{a_n} = \frac{V}{j\pi\zeta_o}\left\{\frac{1}{a_o} + 2\sum_{1}^{\infty} \frac{\cos n\phi}{a_n}\right\} \qquad (9)$$

From this, the impedance of the antenna, Z, is found to be

$$Z = \frac{V}{I(o)} = j\pi\zeta_o \left\{\frac{1}{a_o} + 2\sum_{1}^{\infty} \frac{1}{a_n}\right\}^{-1} \qquad (10)$$

These results, (9) and (10), which were obtained by Hallén, constitute a formal solution of the loop antenna. From them the transmitting pattern and by reciprocity the receiving pattern can be obtained. However, in order to make them useful, some way must be found to evaluate the series numerically.

It can be shown that these equations, (9) and (10), are in agreement with the theory of small loops. Using equation (4), and the explicit evaluation of $K_n$ given in Appendix I, it is readily shown that for loops small in comparison to the wavelength the current is nearly a constant, independent of $\phi$ and

$$kb \ll 1, \qquad Z \cong j\pi\zeta_o a_o = j\pi\zeta_o kbK_1$$

$$\cong \frac{\pi\zeta_o}{6} k^4b^4 + j\pi\zeta_o kb\left[\ln\frac{8b}{a} - 2\right]$$

This is the usual formula for the resistance and reactance of a small loop.

## III

## The Fourier Series

It is apparent from the preceding derivation that the usefulness of this method of solution depends on the evaluation of the series

$$I(\phi) = \frac{jV}{\pi\zeta_o}\left\{\frac{1}{a_o} + 2\sum_1^{\infty}\frac{\cos n\phi}{a_n}\right\}. \qquad (11)$$

Hallén proved that, for large n, the coefficients approach asymptotically the value

$$a_n \sim -\frac{n^2}{\pi kb}\left\{\ln\frac{2b}{a} - \gamma - \ln n\right\},$$

where $\gamma$ (= .5772) is Euler's constant. It is apparent that $a_n$ becomes extremely small for values of n such that

$$n \cong n_o = \frac{2b}{a}\; e^{-\gamma}$$

Hence the series (11) has a "singularity" near $n \cong n_o$. From this fact Hallén concluded that the series (11) could only be used in an "asymptotic" fashion, i.e., it must converge satisfactorily by $n \cong \frac{n_o}{2}$ since after this the value of the individual terms begin increasing in magnitude. This restriction meant that the series solution (11) was only useful for kb small or b/a very large. Even with this limitation the summation of $n_o/2$ terms of this series in a formidable task and, at best, yields relatively inaccurate results because of the "singularity."

It must be remembered at this point that current is both bounded and continuous (for physical reasons) and hence the series (11) must converge. Adopting this point of view, the problem then becomes one of treating Hallén's "singularity" in a more rigorous fashion.

A derivation of the value of $a_n$ is given in Appendix A, which is essentially identical to that of Hallén's, but which includes the dominant complex term as well. The result for large n is

$$a_n \sim \frac{1}{\pi}\left(kb - \frac{n^2}{kb}\right)\left(\ln n_o - \ln n - j\,\frac{(kb)^{2n+1}}{\Gamma(2n+2)}\right) \qquad \begin{matrix} n > kb \\ n \gg 1 \end{matrix} \ ,$$

where $n_o = \frac{2b}{a} e^{-\gamma}$.

It is apparent that the inclusion of the rather negligible complex term in (12) cannot alter significantly the sum of the resulting series. However, with its inclusion $a_n$ is never equal to zero. This fact will be used subsequently to permit a replacement of the series by an integral.

The following work will be restricted to loops in which $kb \leq 2.5$ — i.e., the circumference of the loop is less than two-and-a-half wavelengths.* Almost all loop antennas of practical interest are contained in this range. The series (11) can then be written in the form

$$I(\phi) = \frac{V}{j\pi \zeta_o}\left\{\frac{1}{a_o} + 2\sum_{1}^{4}{}' \frac{\cos n\phi}{a_n} + \psi(\phi)\right\} , \qquad (12)$$

where

$$\psi(\phi) = 2\sum_{5}^{\infty}{}' \frac{\cos n\phi}{a_n} . \qquad (13)$$

The procedure to be used will sum the first five terms of the series explicitly, and replace the remainder of series $\psi(\phi)$ by an integral.

Now, it can be shown by an insertion of numerical values that for

------

*The derivation can readily be modified to include values of kb larger than 2.5 if desired.

$kb \leq 2.5$ and $n \geq 5$, the value of $a_n$ differs negligibly from the asymptotic value given by (12). Hence, to an excellent approximation,

$$\psi(\theta) = -2\pi kb \int_{5}^{\infty} \frac{\cos n\phi}{(n^2 - k^2b^2)[\ln n_o - \ln n - j(kb)^{2n+1}/\Gamma(2n+2)]} . \quad (14)$$

The series (14) will now be replaced by an integral. The particular formula to be used is

$$\sum_{N}^{\infty} a_n = \int_{N-\frac{1}{2}}^{\infty} a_x\, dx + \sum_{o}^{\infty} c_n \left[\frac{d^{2n+1}}{dx^{2n+1}} a_x\right]_{x=N-\frac{1}{2}} , \quad (15)$$

where $c_o = 1/24, c_1 = -\dfrac{7}{2^4 \cdot 360}$, etc.

This result, (15), is valid provided $a_n$ is an analytic function of n in a region which includes the real axis for $n > N - \frac{1}{2} - \epsilon$. Results similar to (15) have been given by Gumowski [5] and others. It is essentially a modification of the Euler-McClaurin sum formula.

Using (15) in connection with (14) yields

$$\psi(\phi) = -2\pi kb \int_{4.5}^{\infty} \frac{\cos x\phi\, dx}{(x^2-k^2b^2)(\ln n_o - \ln x - j\frac{(kb)^{2x+1}}{\Gamma(2x+2)})} \quad (16)$$

$$\frac{2\pi kb}{24}\left[\frac{d}{dx}\frac{\cos x\phi}{(x^2-k^2b^2)(\ln n_o - \ln x - j\frac{(kb)^{2x+1}}{\Gamma(2x+2)})}\right]_{x=4.5} - \cdots$$

This replacement of the series (15) by the integral is possible only because of the complex term, which makes the argument an analytic function of x along the real axis. Since $kb \leq 2.5$, the first (and higher) derivative correction terms in (16) are small (less than 1%) compared to $\psi(\phi)$ and can be ignored, since $\psi(\phi)$ is at best a minor part of $I(\phi)$ in (14). Hence,

$$\psi(\phi) = -2\pi kb \int_{4.5}^{\infty} \frac{\cos x\phi\, dx}{x^2-k^2b^2)(\ln n_o - \ln x - j\frac{(kb)^{2x+1}}{\Gamma(2x+2)})} \quad (17)$$

Next, it can readily be shown that the complex term in the integral of (17) can also be ignored. This yields

$$\psi(\phi) = -2\pi kb \int_{4.5}^{\infty} \frac{\cos x\phi \, dx}{(x^2-k^2b^2)(\ln n_o - \ln x)} . \tag{18}$$

The integral in (18), which is to be interpreted in the "principal value" sense, can be rewritten as follows:

$$\psi(\phi) = \psi_1(\phi) + \psi_2(\phi) \tag{19a}$$

$$\psi_1(\phi) = -2\pi kb \int_{4.5}^{\infty} \frac{\cos x\phi}{\ln n_o - \ln x} \cdot \frac{dx}{x^2} \tag{19b}$$

$$\psi_2(\phi) = -2\pi kb \int_{4.5}^{\infty} \frac{\cos x\phi}{\ln n_o - \ln x} \cdot \frac{k^2b^2 dx}{x^2(x^2-k^2b^2)} \tag{19c}$$

Since $n_o$ is quite large and $kb \leqq 2.5$, (19c) becomes, to a satisfactory approximation:

$$\psi_2(\phi) \cong \frac{-2\pi kb}{\ln n_o - \ln 4.5} \int_{4.5}^{\infty} \frac{k^2b^2 \cos x\phi}{x^2(x^2-k^2b^2)} dx$$

$$\cong \frac{-2\pi k^3b^3}{\ln(\frac{n_o}{4.5})} \int_{4.5}^{\infty} \frac{\cos x\phi}{x^4} dx$$

$$= -\frac{2\pi}{\ln(\frac{n_o}{4.5})} \cdot (\frac{kb}{4.5})^3 \; J_2(\phi)$$

This integral, $J_2(\phi)$, can be evaluated explicitly in terms of sines, cosines, and integral sines.

Using these results, an explicit formula for the current distribution can be written as:

$$I(\phi) = \frac{V}{j\pi\zeta_0}\left\{\frac{1}{a_0} + 2\sum_{1}^{4}\frac{\cos n\phi}{a_n} - \frac{2\pi}{\ln\left(\frac{n_0}{4.5}\right)}\left[\left(\frac{kb}{4.5}\right)J_1(\phi) + \left(\frac{kb}{4.5}\right)^3 J_2(\phi)\right]\right\} \tag{20}$$

where

$$J_1(\phi) = \int_1^{\infty} \frac{\ln\left(\frac{n_0}{4.5}\right)}{\ln\left(\frac{n_0}{4.5}\right) - \ln x} - \frac{\cos(4.5\phi)}{x^2}\, dx \tag{21a}$$

$$J_2(\phi) = \int_1^{\infty} \frac{\cos(4.5x\phi)}{x^4}\, dx \tag{21b}$$

$n_0 = \frac{2b}{a}e^{-\gamma}$ and explicit formulas for the $a_n$ are given in Appendix A.

Note that the $J_k(\phi)$ integrals have only appreciable values near $\phi = 0$. (An asymptotic formula for them, when $\phi > 1$, can readily be obtained.) They cannot be approximated satisfactorily by a sinusoid and are a partial explanation of why approximate methods of dealing with the loop antenna do not yield good quantitative results.

The formula for the impedance of the loop antenna becomes

$$Z = j\pi\zeta_0\left\{\frac{1}{a_0} + 2\sum_{1}^{4}\frac{1}{a_n} - \frac{2\pi}{\ln\left(\frac{n_0}{4.5}\right)}\left[\left(\frac{kb}{4.5}\right)J_1(o) + \left(\frac{kb}{4.5}\right)^3 J_2(o)\right]\right\}^{-1} \tag{22}$$

This result, in connection with Appendix A, forms the basis for the impedance tables presented in Appendix B. The quantities $J_k(o)$ are explicitly given by

$$J_1(o) = \frac{\ln\left(\frac{n_0}{4.5}\right)}{\left(\frac{n_0}{4.5}\right)} \int_{-\infty}^{\ln\left(\frac{n_0}{4.5}\right)} \frac{e^{+x}}{x}\, dx$$

$$J_2(o) = 1/3$$

## IV

## Results

The impedance of loop antennas for various values of b/a have been calculated using equation (22). As a parameter, the quantity

$$\Omega = 2\ln\frac{2\pi b}{a} \tag{23}$$

has been chosen. Note that $2\pi b/a = c/a$, where c is the circumference of the antenna. Hence (23) represents a definition analogous to that used for dipole antennas.

In Appendix B, values of the impedance are tabulated for $0 \leq kb \leq 2.5$ and $\Omega = 8, 9, 10, 11, 12$. They are also presented in graphical form. These impedances are useful for examining the operation of a loop antenna as a function of frequency. For laboratory purposes, however, it is sometimes convenient to have tables available appropriate to holding the frequency fixed and varying the size of the antenna. These are given at the end of Appendix B and have been obtained by interpolation from the earlier tables.

It is perhaps worth while to comment on some of the more obvious features of these loop antenna impedances. As can be seen, the first antiresonance, occurring when the circumference of the loop approximates a half-wavelength, is extremely sharp. This well-known effect is easily explained by noting that a sufficiently small loop resembles closely a short-circuited quarter-wavelength transmission line and has a correspondingly sharp antiresonance.

Of equal interest is the rapid disappearance of resonances as the circumference of the antenna increases. Thus, for $\Omega \leq 9$, a second resonance point does not even exist. If one compares these impedances with those for a dipole antenna, it is seen that the two are similar, both qualitatively and quantitatively, for $c > \lambda$. The prime difference is that the loop is essentially more capacitive (by about 130 ohms) than a dipole. This can be explained on the basis that charged surfaces are closer together on a loop than on a dipole. This shift in reactance level by 130 ohms permits the dipole to have several resonances and antiresonances, whereas, as noted previously, a moderately thick loop ($\Omega<9$) has essentially only one anti-resonance. The resistance curves for the loop and dipole are very similar, with the resistance minima having almost identical values.

It is interesting to compare these theoretical loop impedances with some experimentally measured ones. Miss Phyllis Kennedy of Cruft Laboratory has measured some loop impedances, using a half-loop over an image plane, and driven by a two wire line. The explicit configuration is indicated in Fig. 4.1a. One set of the admittances measured by Miss Kennedy appears in Fig. 4.1b together with the corresponding theoretical curves. The agreement between the theoretical and experimental curves is seen to be excellent. It is seen that the resistance peaks near resonance on the theoretical conductance curves are slightly higher than those on the experimental curve. This could have been anticipated as ohmic losses of the loop were not taken into account in the theoretical solution. The two susceptance curves differ by a slight additive amount throughout the entire range. This can readily be attributed to the so-called end coupling effect of the feeding line, which arises from the fact that the transmission-line excitation differs from the "slice generator" used in the theoretical model. King [6] has calculated this end effect for a dipole antenna. The dominant correction term is a negative capacitance in shunt with the antenna. Quite obviously, the end correction for a loop antenna should be similar, even to the order of magnitude. If such an approximate correction is made to the susceptance curve of Fig. 4.1b, this is changed in the right direction.

In Appendix C, values of the quantities $1/a_k$ and the functions $J_k(\phi)$ are presented graphically to facilitate evaluation of the current distribution using equations (20) and (21). To obtain an idea of the type of current distributions on loop antennas, some were calculated for the explicit case of $\Omega = 10$. Owing to the fact that the $J_k(\phi)$ were evaluated by numerical integration, there exists a slight discrepancy between $I(o)$ and the admittance. Since the admittance values are more accurate, they were used in place of $I(o)$.

One of the classic assumptions in antenna literature is that a small loop has a constant current distribution. To examine the validity of the assumption, the actual current distribution were calculated for $\Omega = 10$ and kb = .1, .2, .3, and .4. These appear in Fig. 4.2. It is apparent that for the smallest loop, kb = .1, the current varies in magnitude by about 5 and hence can be considered reasonably constant. For kb = .2, however, the variation is well over 10 %. On the basis of these results, one would be led to the conclusion that loops much larger than kb = . 2 cannot be considered small.

In order to obtain an idea of how the distribution of current varies as the size loop increased, values of it were calculated for $\Omega = 10$ and kb = .5, 1.0, 1.5, 2.0, and 2.5. These results appear in Fig. 4.3. Perhaps the most noticeable feature in these curves appears in the plots of magnitude and phase for the larger values kb. For values of $\phi < 90^{\circ}$, it is apparent that the current distribution is beginning to approximate a traveling wave, in the sense that variations in the magnitude have been reduced and the phase is becoming linear. This is in agreement with the observation made in connection with the impedances, namely, that for larger kb the magnitude of the variation of the resistance is reduced.

## V
## Acknowledgements

The author wishes to thank Professor R.W.P. King of Harvard University for his help and encouragement with this research, Miss Phyllis Kennedy for making her measurements available; and to Mr. Leon Levy, who performed the numerical computations.

The author also wishes to acknowledge the financial support he received from the Atomic Energy Commission as a Fellow during the early phase of this research, and during the latter part, under the sponsorship of the Office of Naval Research, the Signal Corps of the U.S. Army, and the U. S. Air Force (Contract N5ori-76).

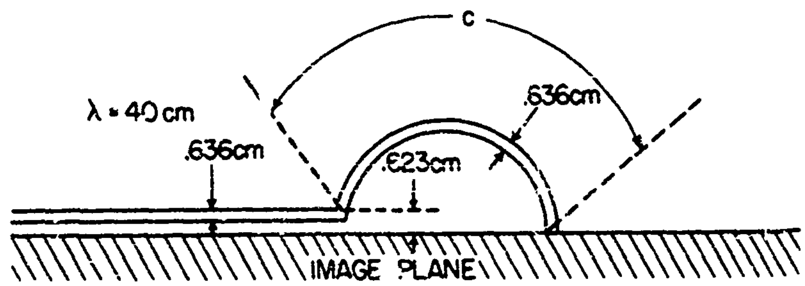



FIG. 4.1a EXPERIMENTAL DIMENSIONS

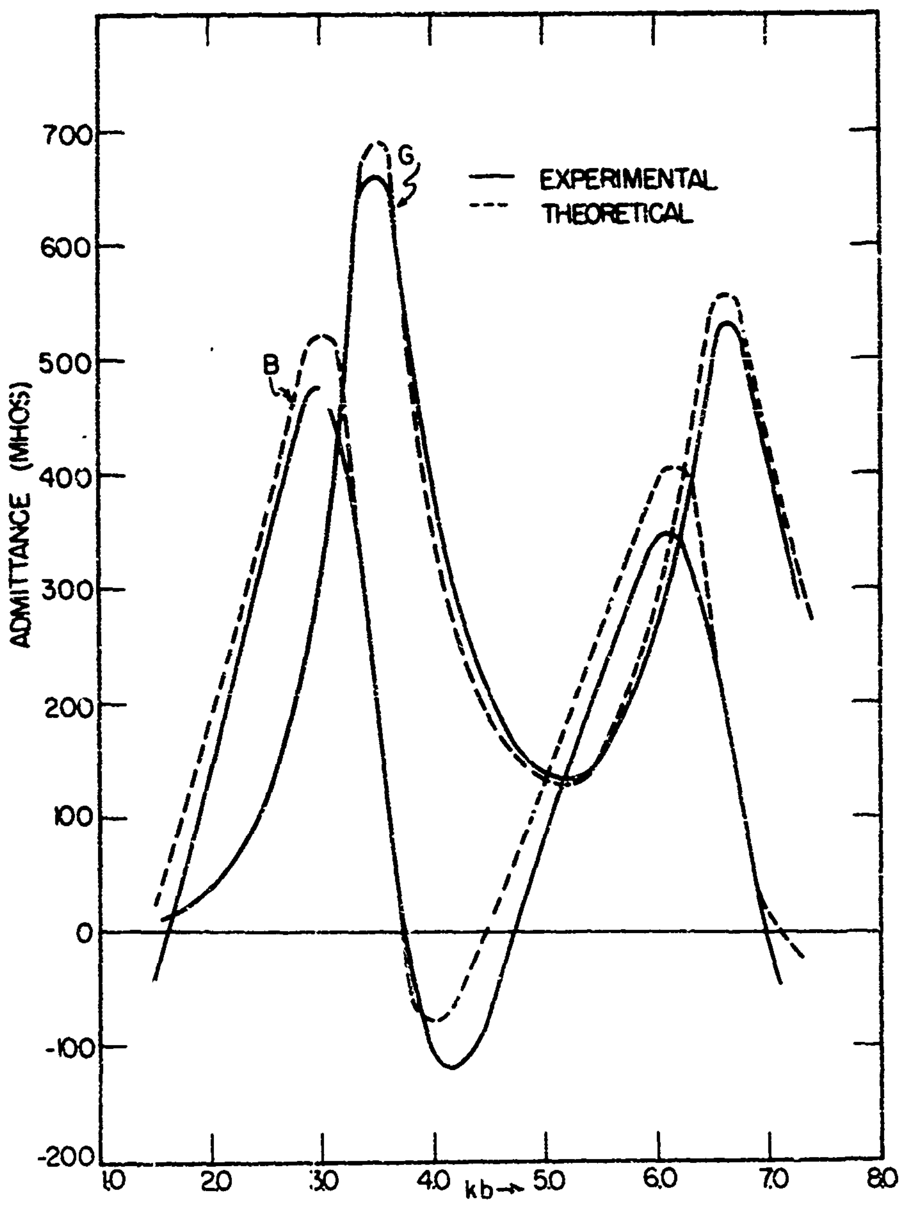



FIG. 4.1b

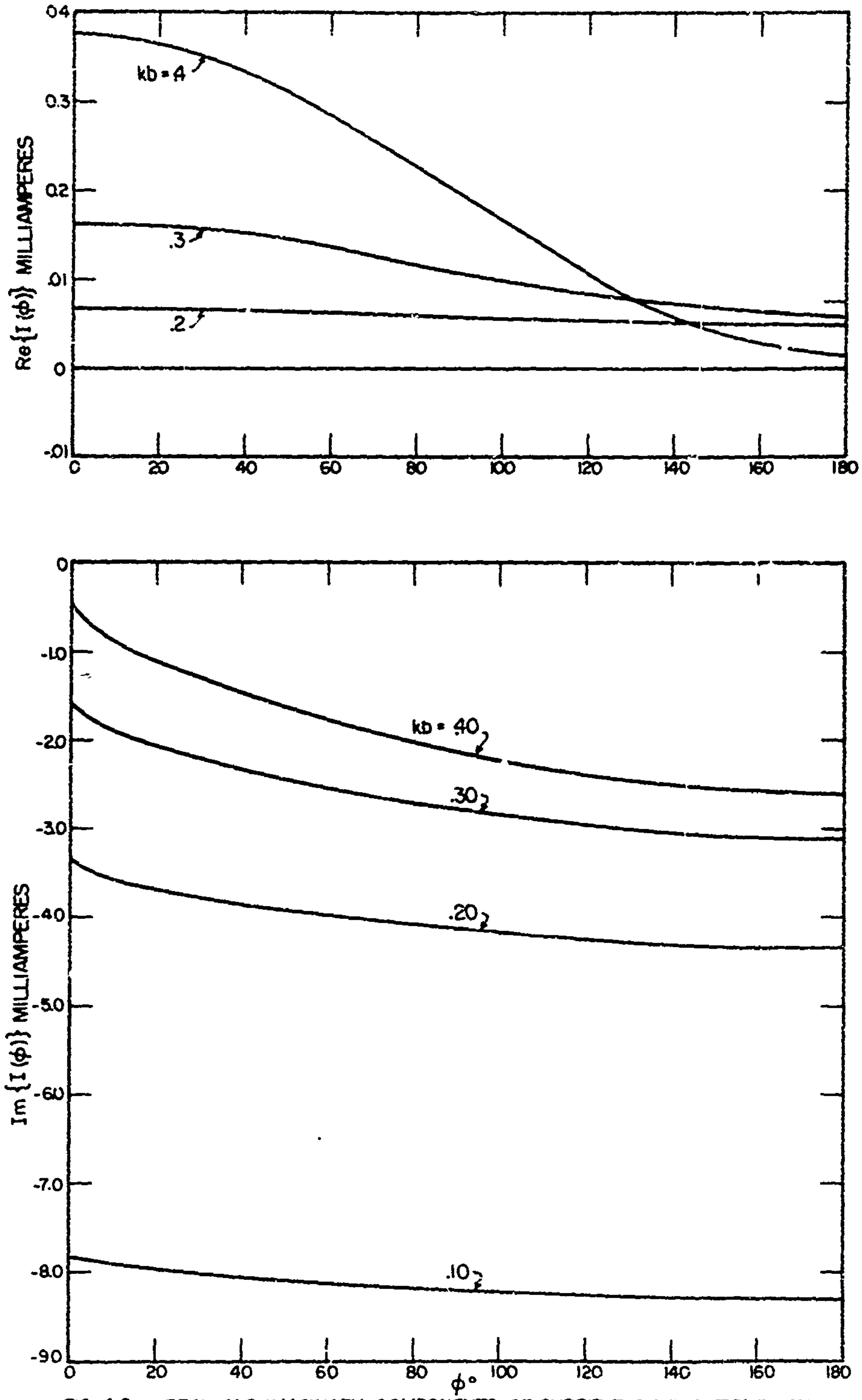



FIG 4.2a REAL AND IMAGINARY COMPONENTS OF CURRENT DISTRIBUTIONS ON SMALL LOOP ANTENNAE

$\Omega = 2\ln(2\pi b/a) = 10$

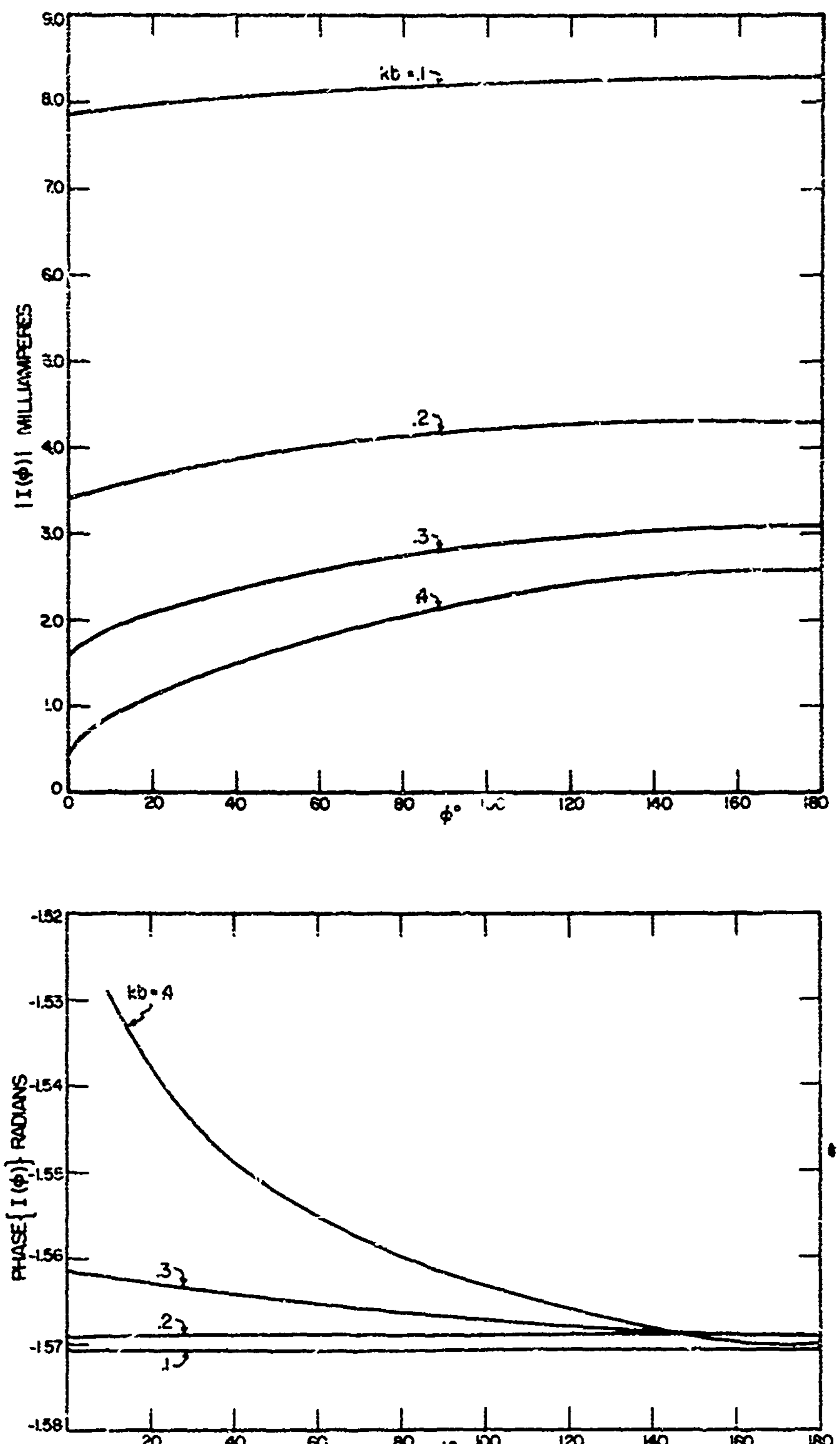



FIG. 4.2b MAGNITUDE AND PHASE OF CURRENT DISTRIBUTIONS ON SMALL LOOP ANTENNAE

$\Omega = 2\ln(2\pi b/a) = 10$

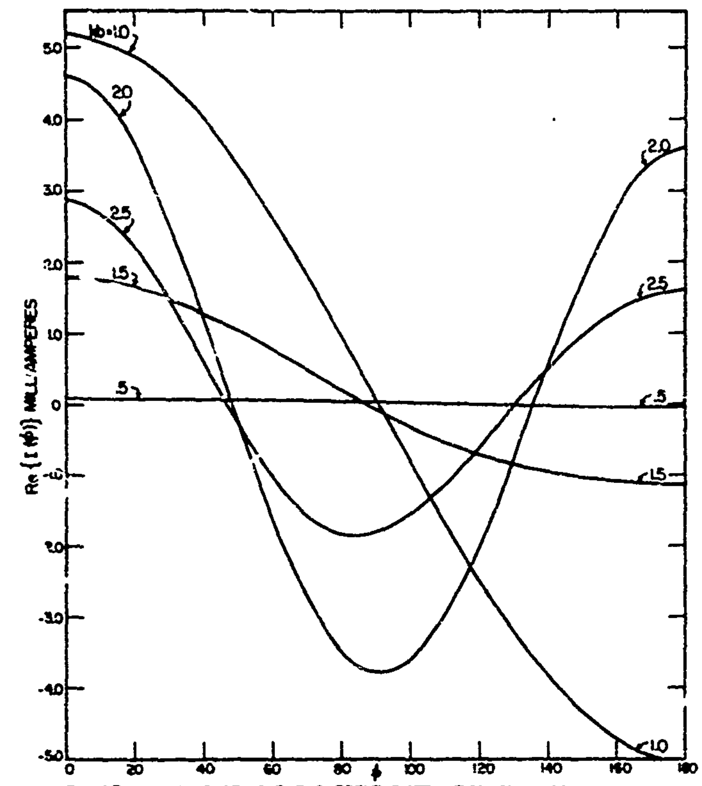



FIG. 4.3a REAL PART OF THE CURRENT DISTRIBUTION ON LOOP ANTENNAE

$\Omega = 2\ln(2\pi b/a) = 10$

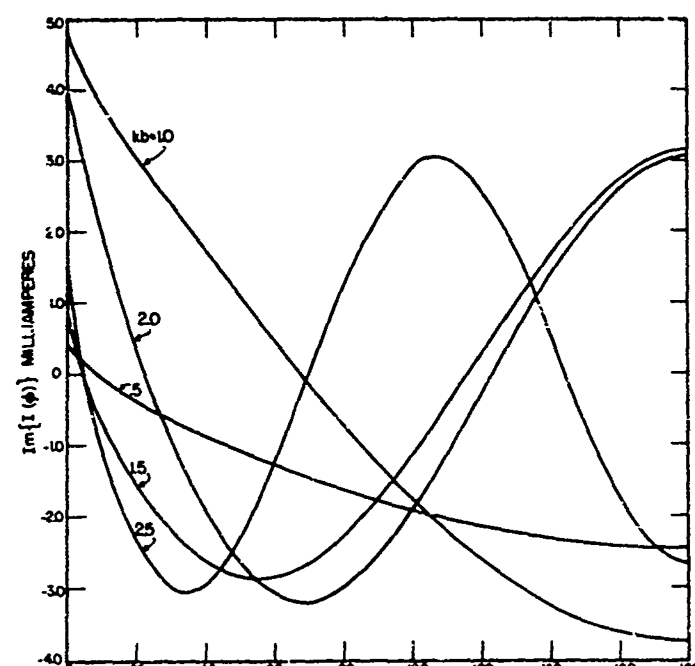



FIG. 4.3b IMAGINARY COMPONENT OF CURRENT DISTRIBUTION ON LOOP ANTENNAE

$\Omega = 2\ln(2\pi b/a) = 10$

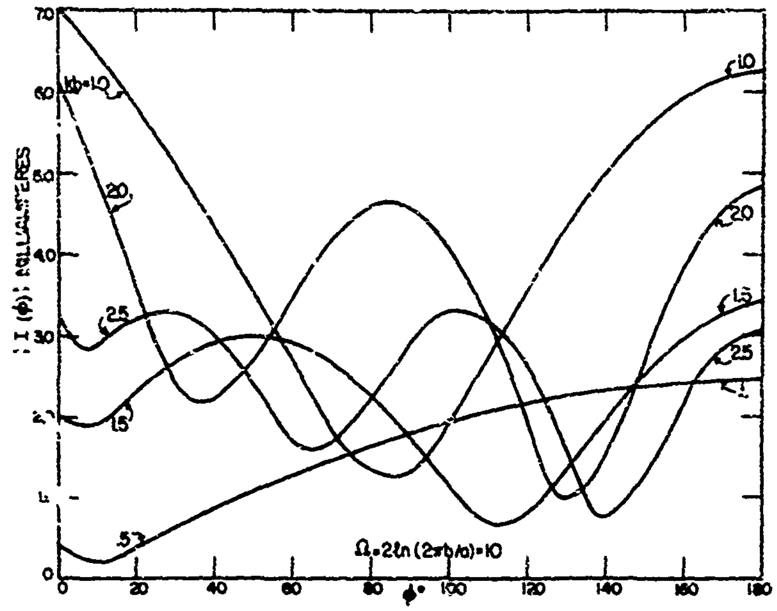



FIG. 4.3c MAGNITUDE OF CURRENT DISTRIBUTION ON LOOP ANTENNAE

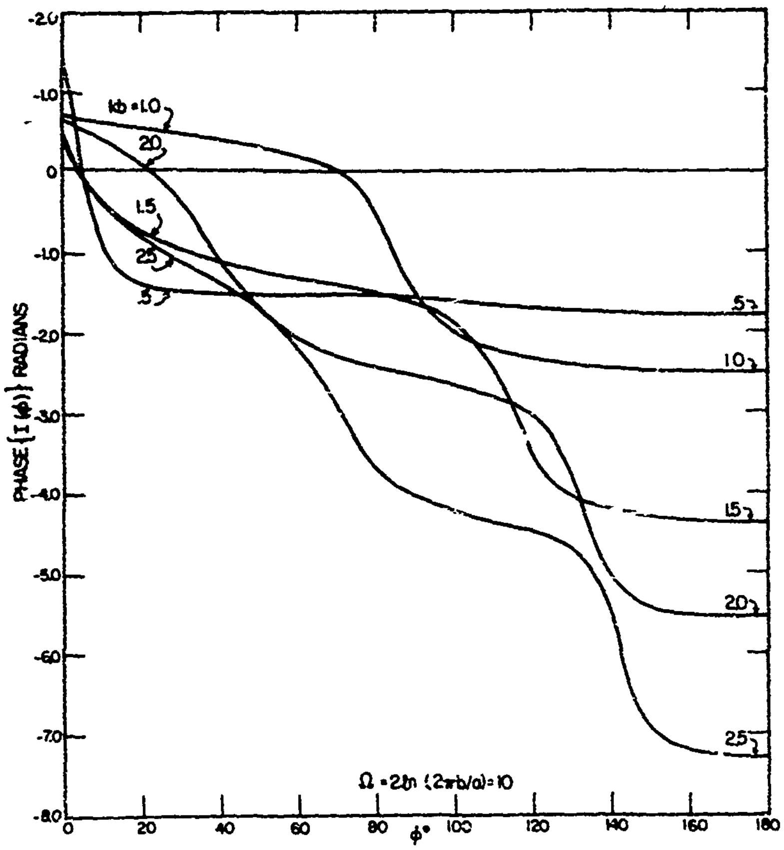



FIG. 4.3d PHASE OF CURRENT DISTRIBUTION ON LOOP ANTENNAE

## References

1. H. C. Pocklington, Proc. Cambridge Phil. Soc. 9, 324 (1897).

2. E. Hallén, Nova Acta Reg. Soc. Uppsala, Ser. IV, vol. II, no. 4, November, 1938.

3. Tung Chang, Technical Report No. 16, Cruft Laboratory, Harvard University.

4. S. A. [illegible]unoff and H. T. Friis, Antenna Theory and Practice, Sect. 13.12; John Wiley, New York, 1952.

5. I. Gumowski, "Summation of Slowly Converging Series," J. Appl. Phys. 24, 1068 (1953).

6. R. King, "Theory of Antennas Driven from Two-Wire Line," J. Appl. Phys. 20, 832 (1949).

## Appendix A

## Evaluation of $K_n$

From (4) it is seen that

$$A_n = K_{n+1} - K_n = \frac{1}{2\pi} \int_0^{2\pi} \frac{e^{-jkbR(\phi)}}{R(\phi)} [e^{j(n+1)\phi} - e^{jn\phi}] d\phi \qquad n > 0$$

$$= \frac{1}{2\pi} \int_0^{2\pi} \frac{e^{-jkbR(\phi)}}{R(\phi)} e^{j(n+\frac{1}{2})\phi} \; 2j \sin \phi/2 \; d\phi$$

The "thin-wire" approximation is that $k^2a^2 << 1$, $a^2 << b^2$. Neglecting terms of this order of magnitude yields

$$A_n = \frac{1}{2\pi} \int_0^{2\pi} \frac{e^{-2jkb \sin \phi/2}}{2 \sin \phi/2} e^{j(n+\frac{1}{2})\phi} \; 2j \sin \phi/2 \; d\phi$$

$$+ \text{ term of order } (a^2/b^2)$$

$$= \frac{j}{\pi} \int_0^{\pi} e^{-2jkb \sin \theta + j(2n+1)\theta} d\theta$$

$$= j \left\{ J_{2n+1}(2kb) - j\Omega_{2n+1}(2kb) \right\}$$

where

$$J_{2n+1}(x) = \frac{1}{\pi} \int_0^{\pi} \sin (x \sin \theta - (2n+1)\theta) d\theta$$

is the Bessel function of order 2n+1, and

$$\Omega_{2n+1}(x) = \frac{1}{\pi} \int_0^{\pi} \cos (x \sin \theta - (2n+1)\theta) d\theta$$

is the Lommel-Weber function of order 2n+1, tabulated in Janke-Emde.

Thus, the above result provides a reversion formula for $K_n$, i.e.,

$$A_n = K_{n+1} - K_n = \Omega_{2n+1}(2kb) + j\, J_{2n+1}(2kb), \qquad n > 0$$

Therefore, all that remains to evaluate is $K_o$. This coefficient can be written as

$$K_o = \frac{1}{2\pi}\int_0^{2\pi} \frac{e^{-jkbR(\phi)}}{R(\phi)}\, d\phi$$

$$= \frac{1}{2\pi}\int_0^{2\pi} \frac{e^{-jkbR(\phi)}-1}{R(\phi)}\, d\phi + \frac{1}{2\pi}\int_0^{2\pi} \frac{d\phi}{R(\phi)}$$

Now,

$$\frac{1}{2\pi}\int_0^{2\pi} \frac{e^{-jkbR(\phi)}-1}{R(\phi)}\, d\phi = \frac{1}{2\pi}\int_0^{2\pi} \frac{e^{-2jkb\sin\phi/2}-1}{2\sin\phi/2}\, d\phi + \text{term } o(a^2/b^2)$$

$$= \int_0^{2kb} dx \left\{ -\frac{j}{\pi}\int_0^{\pi} e^{-jx\sin\phi}\, d\phi \right\}$$

$$= -1/2\int_0^{2kb} \Omega_o(x)dx - j/2\int_0^{2kb} J_o(x)dx$$

It also can be shown[2]

$$\frac{1}{2\pi}\int_0^{2\pi} \frac{d\phi}{R(\phi)} = \frac{1}{\pi}\ln\frac{8b}{a} + \text{terms } o(a^2/b^2)$$

So, to the order of approximation consistent with original integral equation,

$$K_o = \frac{1}{\pi}\ln\frac{8b}{a} - 1/2\int_0^{2kb} \Omega_o(x)dx - j/2\int_0^{2kb} J_o(x)dx$$

$$A_n = K_{n+1} - K_n = \Omega_{2n+1}(2kb) + j\ J_{2n+1}(2kb)$$

Another expression, useful for determining $K_n$ for large n, can also be found. From the above it is seen that

$$K_n = K_o + \sum_{o}^{n-1} A_n$$

Inserting the integral expressions for $A_n$ yields

$$K_n = K_o + \frac{j}{\pi} \sum_{o}^{n-1} \int_o^{\pi} e^{-2jkb\sin\phi + j(2n+1)\phi} \, d\phi$$

$$= K_o + \frac{j}{\pi} \int_o^{\pi} e^{-2jkb\sin\phi} \left\{ \frac{e^{2jn\phi} - 1}{\sin\phi} \right\} d\phi$$

Inserting the value for $K_o$,

$$K_n = \frac{1}{\pi} \ln \frac{8b}{a} + \frac{1}{2\pi} \int_o^{\pi} [e^{-2jkb\sin\phi + 2jn\phi} - 1] \frac{d\phi}{\sin\phi}$$

$$= \frac{1}{\pi} \ln \frac{8b}{a} + \frac{1}{2\pi} \int_o^{\pi} [e^{-2jkb\sin\phi} - 1] \frac{e^{2jn\phi}}{\sin\phi} d\phi$$

$$+ \frac{1}{2\pi} \int_o^{\pi} [e^{2jn\phi} - 1] \frac{d\phi}{\sin\phi}$$

$$= \frac{1}{\pi} \ln \frac{8b}{a} - \frac{1}{2} \int_o^{2kb} [\Omega_{2n}(x) + j\, J_{2n}(x)] dx - \frac{2}{\pi} \sum_{o}^{n-1} \frac{1}{2K+1}$$

This result can be used conveniently to determine the form of $K_n$ for large N. For $n \gg kb$, the integral is small, vanishing in the limit. Thus

$$n \gg 1; \quad n > kb, \quad K_n \sim \left( \frac{1}{\pi} \ln \frac{8b}{a} - \frac{2}{\pi} \sum_{o}^{n-1} \frac{1}{2K+1} \right) - \frac{j}{2} \int_o^{2kb} J_{2n}(x) dx$$

Now, using Sterling's formula to evaluate the harmonic series, it can be shown that

$$\sum_{o}^{n-1} \frac{1}{2K+1} = \frac{\gamma}{2} + \frac{1}{2} \ln 4n, \ \gamma (= .5772) \text{ is Euler's constant.}$$

Similarly, for $n > kb$

$$J_n(x) \cong \frac{1}{\Gamma(n+1)}\left(\frac{x}{2}\right)^n$$

So $K_n \sim \frac{1}{\pi}\left(\ln \frac{2b}{a} - \gamma - \ln n\right) - j\,\frac{(kb)^{n+1}}{\Gamma(2n+2)}$ $\quad\begin{cases} n^2 \gg 1 \\ n > kb \end{cases}$

The Fourier coefficient, $a_n$, (4), can be written as

$$a_n = \left(kb - \frac{n^2}{kb}\right) K_n + kb \left[\frac{A_n - A_n - 1}{2}\right] .$$

Since $A_n$ vanishes for large n, the asymtotic value of $a_n$ is given by

$$a_n \sim \left(kb - \frac{n^2}{kb}\right)\left[\frac{1}{\pi}\left(\ln \frac{2b}{a} - \gamma - \ln n\right) - j\frac{(kb)^{2n+1}}{\Gamma(2n+2)}\right] \qquad \begin{cases} n^2 \gg 1 \\ n > kb \end{cases}$$

Finally, (by simple insertion of numerical values), it can be shown that for $k \leqq 2.5$, and $n \geqq 5$, that the asymtotic value of $a_n$ as given above differs negligibly from the correct value.

## Appendix B

## Input Impedance of Loop Antennas

In the following tables impedances, $Z = R + jX$, are given in ohms and admittances, $Y = 1/Z = G + jB$, are given in mhos. The loop radius is designated by b and the loop wire radius by a. The ratio b/a is expressed in terms of the parameter $\Omega = \ln \frac{2\pi b}{a}$. Note that $2\pi b = c$, the circumference of the loop, and $kb = \frac{2\pi b}{\lambda} = \frac{c}{\lambda}$, where $\lambda$ is the wavelength. Thus kb is simply the circumference of the loop divided by the wavelength.

| | |
|---|---|
| Part I: | Graphs of the Input Impedance as a function of frequency |
| Figure B1: | R vs. kb for $\Omega = 8, 9, 10, 11, 12$; $kb \leq 2.5$ |
| Figure B2: | X vs. kb for $\Omega = 8, 9, 10, 11, 12$; $kb \leq 2.5$ |
| Figure B3: | G vs. kb for $\Omega = 8, 9, 10, 11, 12$; $kb \leq 2.5$ |
| Figure B4: | B vs. kb for $\Omega = 8, 9, 10, 11, 12$; $kb \leq 2.5$ |
| Figure B5: | Locus of Resonance and Anti-Resonance Points |
| Part II: | Tables of Input Impedance and Admittance as a function of frequency. |
| Table B1: | Z and Y vs. kb for $\Omega = 8, 9$; $kb \leq 2.5$ |
| Table B2: | Z and Y vs. kb for $\Omega = 10, 11$; $kb \leq 2.5$ |
| Table B3: | Z and Y vs. kb for $\Omega = 12$; $kb \leq 2.5$ |
| Part III: | Tables of Input Impedance for ka constant |
| Table B4: | Z vs. kb for a = 3/16 in., 1/4 in., 5/16 in. at $\lambda = 100$ cm. |
| Table B5: | Z vs. kb for a = 3/8 in., 1/2 in., 3/4 in., at $\lambda = 100$ cm. |

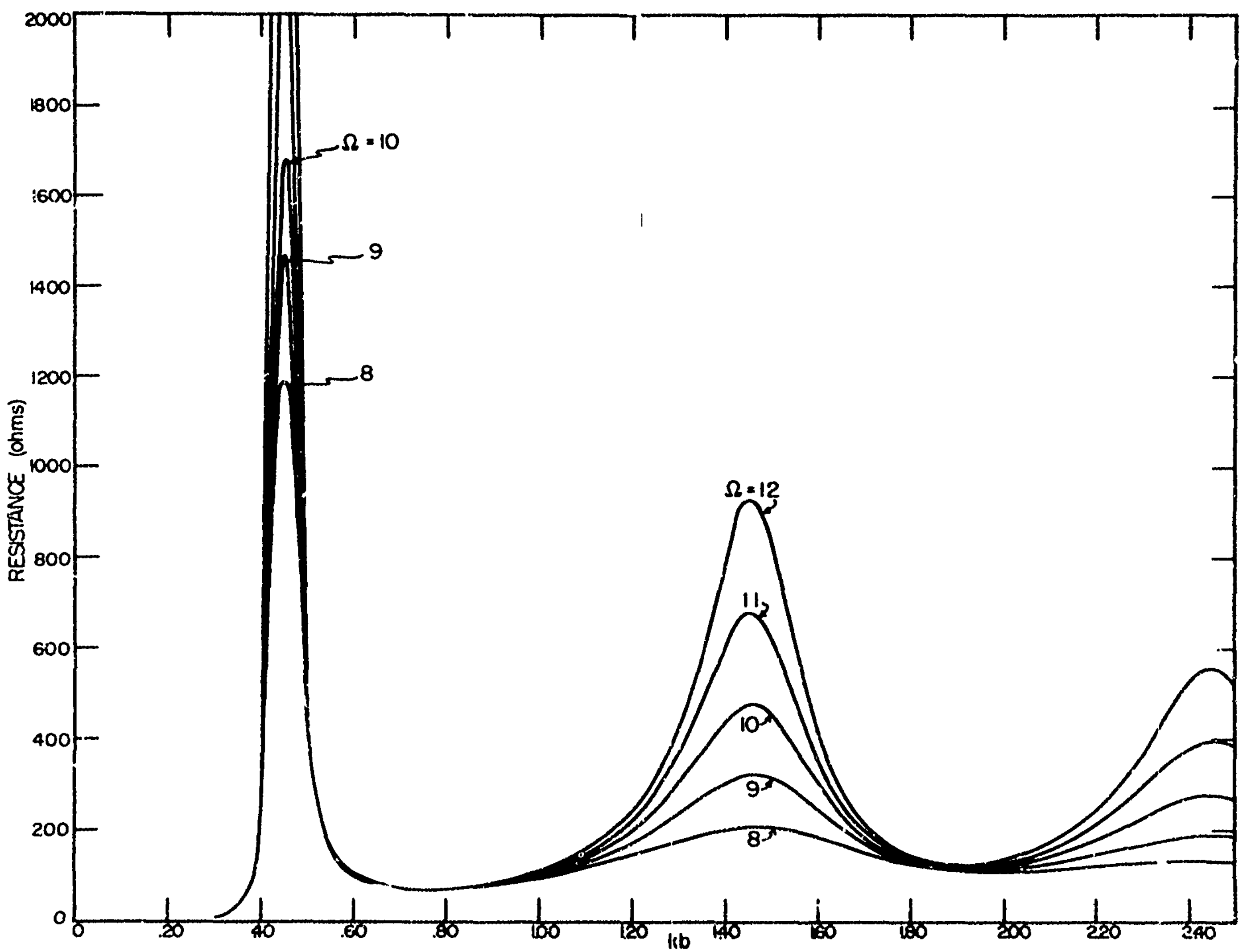



FIG. B1 RESISTANCE OF A LOOP ANTENNA AS A FUNCTION OF FREQUENCY $\Omega = 2\ln(2\pi b/a)$

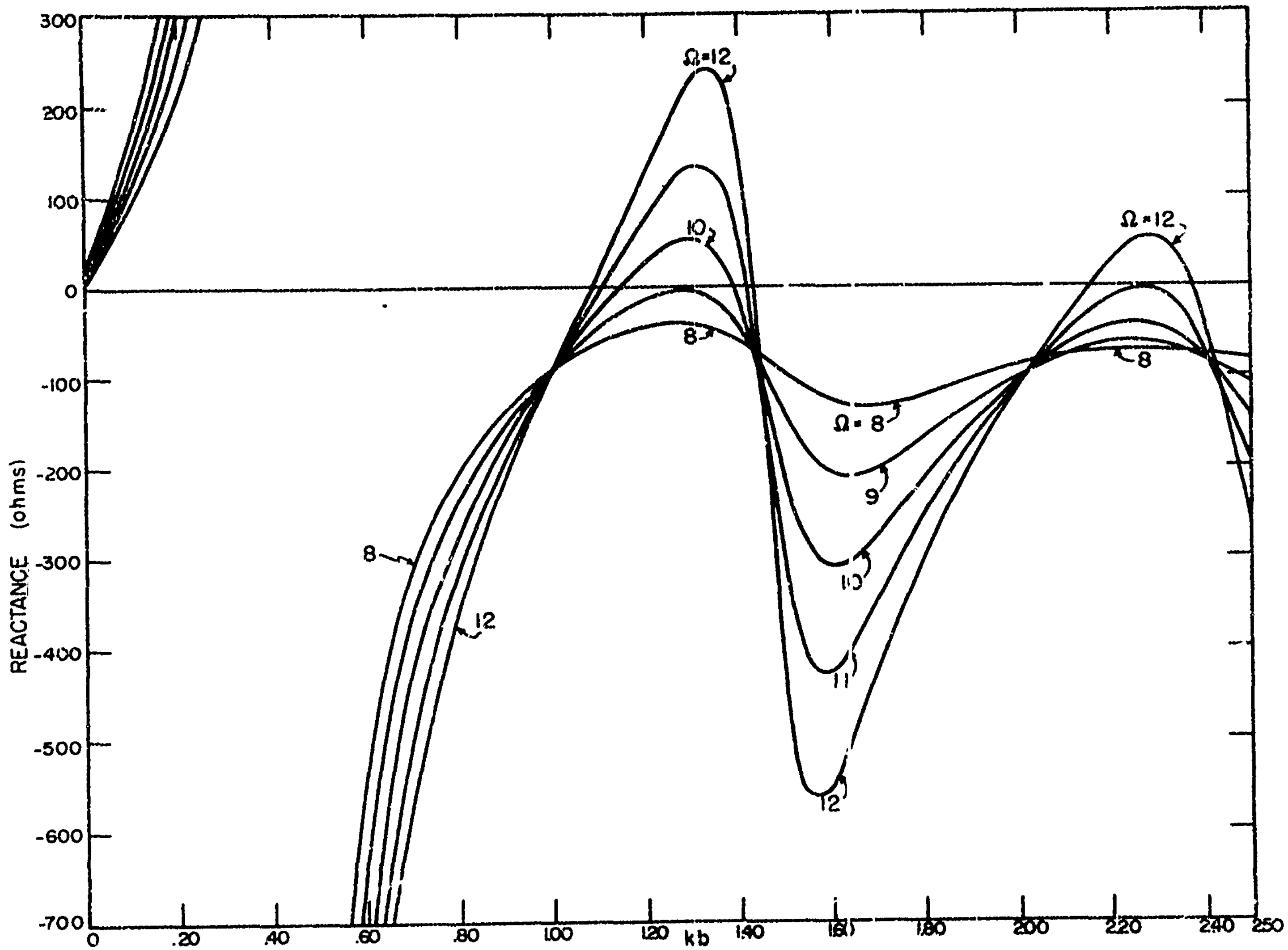



FIG. B2 REACTANCE OF A LOOP ANTENNA AS A FUNCTION OF FREQUENCY $\Omega = 2 \ln(2\pi b/a)$

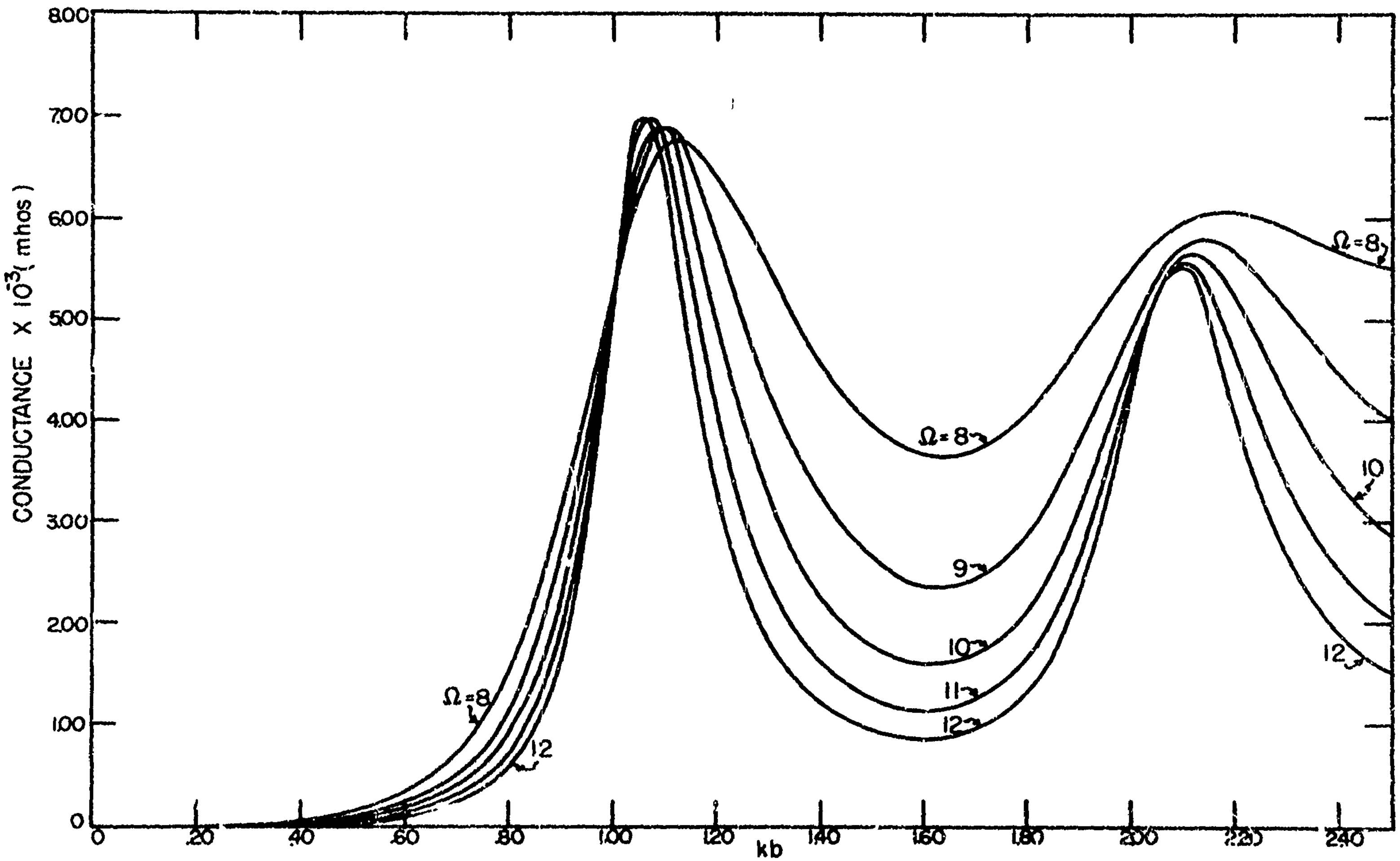



FIG. B3 CONDUCTANCE OF A LOOP ANTENNA AS A FUNCTION OF FREQUENCY $\Omega = 2\ln(2\pi b/a)$

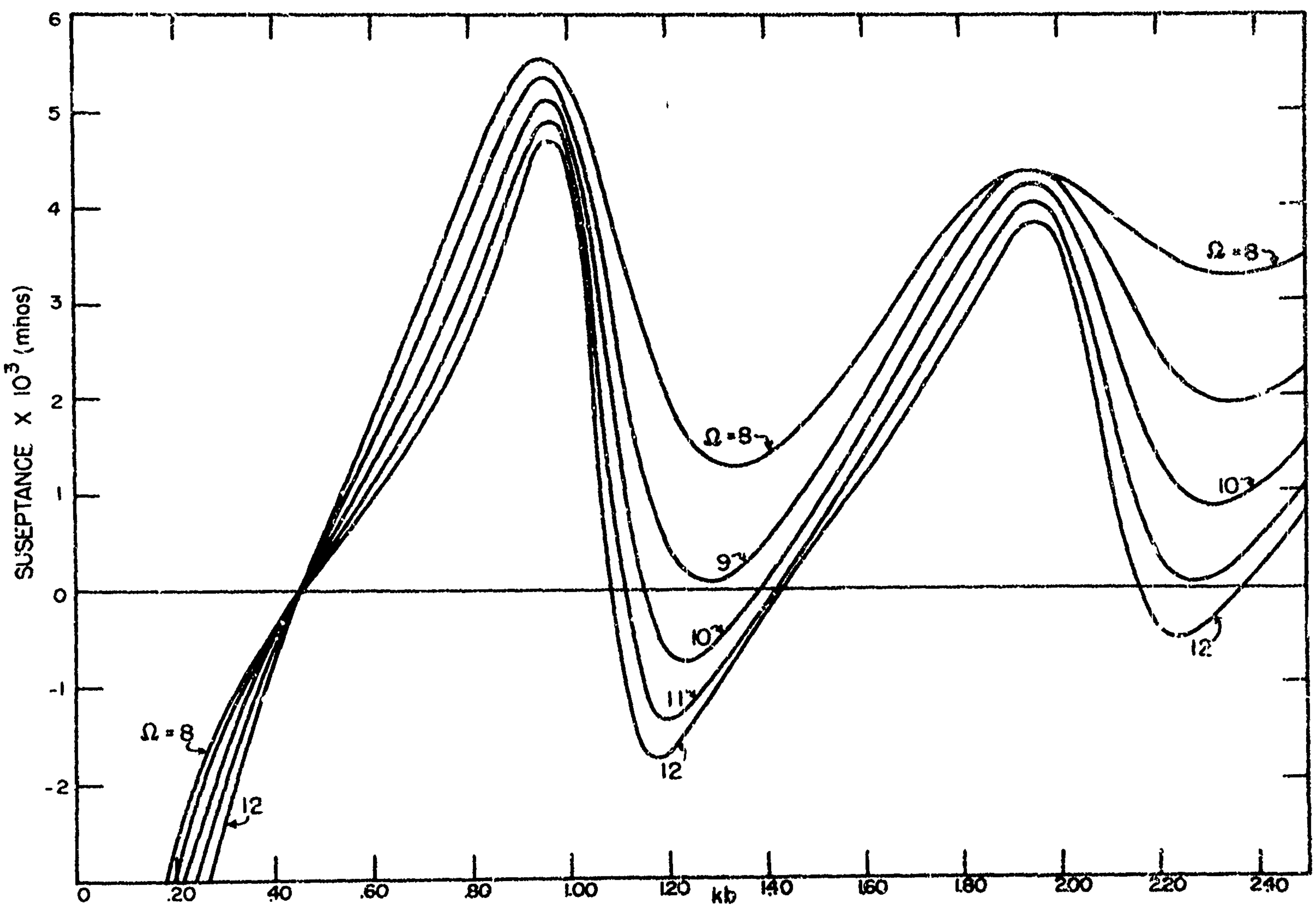
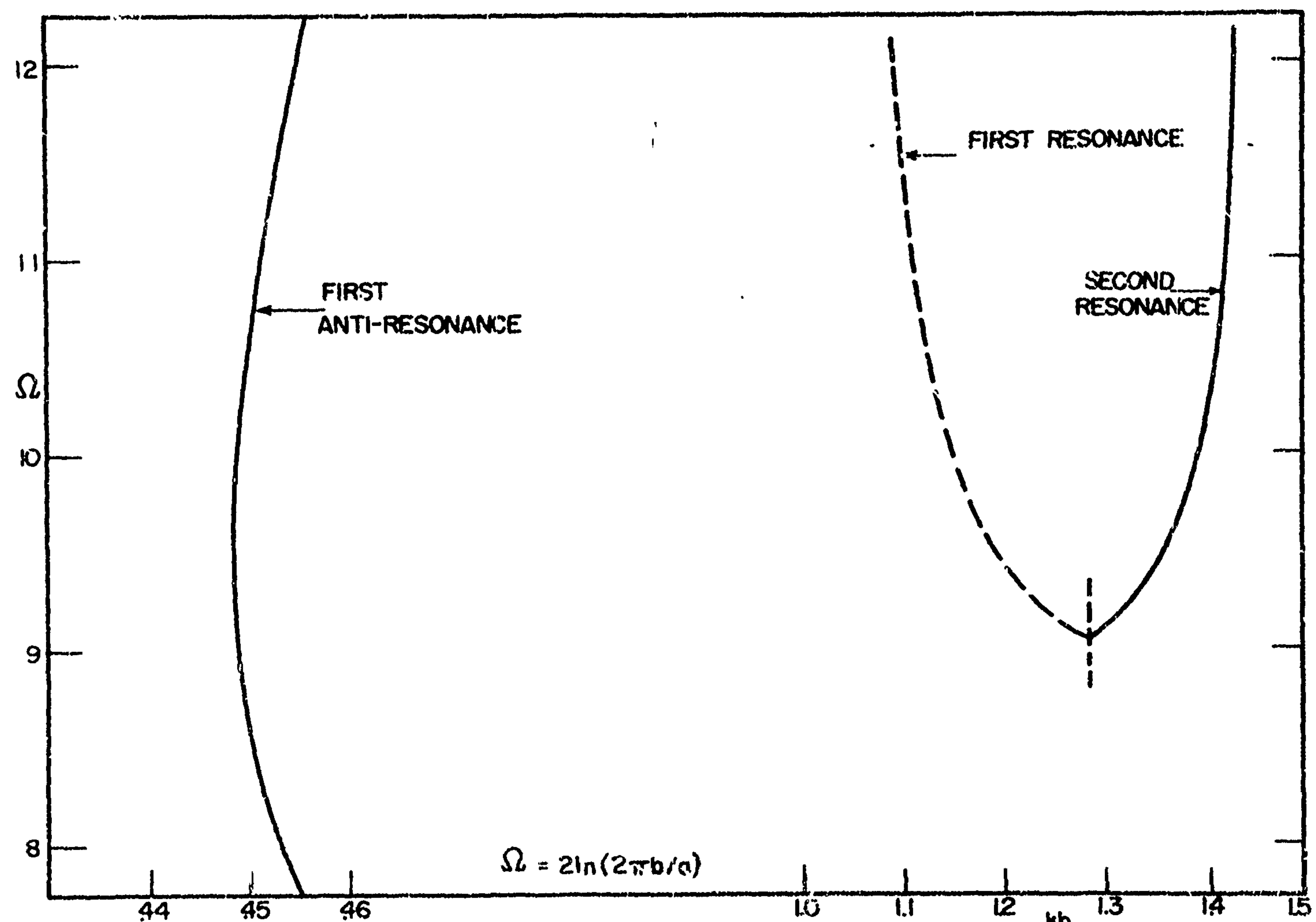



FIG. B4 SUSEPTANCE OF A LOOP ANTENNA AS A FUNCTION OF FREQUENCY $\Omega = 2 \ln(2\pi b/a)$




RESONANCE AND ANTI-RESONANCE POINTS FOR LOOP ANTENNA

FIG. B5

TABLE B1

**Impedance of Loop Antennae**

**as a Function of Frequency**

$\Omega = 8$, $2\pi b/a = 54.60$ (left R, X, G, B columns); $\Omega = 9$, $2\pi b/a = 90.02$ (right R, X, G, B columns)

| R | X | $G \cdot 10^3$ | $B \cdot 10^3$ | kb | R | X | $G \cdot 10^3$ | $B \cdot 10^3$ |
|---|---|---|---|---|---|---|---|---|
| .0048 | 43.57 | .0025 | -22.95 | .05 | .0046 | 51.99 | .0017 | -19.23 |
| .0402 | 88.77 | .0052 | -11.36 | .10 | .0392 | 107.4 | .0034 | - 9.311 |
| .1533 | 140.5 | .0078 | - 7.119 | .15 | .1538 | 172.0 | .0052 | - 5.814 |
| .5939 | 205.7 | .0140 | - 4.860 | .20 | .5917 | 252.2 | .0093 | - 3.964 |
| 1.742 | 293.1 | .0203 | - 3.412 | .25 | 1.756 | 360.6 | .0135 | - 2.773 |
| 6.143 | 427.4 | .0336 | - 2.339 | .30 | 6.327 | 529.1 | .0226 | - 1.890 |
| 23.72 | 675.2 | .0520 | - 1.479 | .35 | 25.57 | 853.2 | .0351 | - 1.171 |
| 140.1 | 1344.0 | .0767 | - .7361 | .40 | 162.5 | 1776.0 | .0518 | - .5662 |
| 7972.7 | 2189.8 | .1172 | - .0322 | .45 | 1188.0 | -3119.4 | .0796 | .0209 |
| 502.2 | -1677.8 | .1638 | .5471 | .50 | 415.0 | -1887.9 | .1111 | .5054 |
| 169.0 | - 824.4 | .2387 | 1.164 | .55 | 154.3 | - 962.0 | .1625 | 1.013 |
| 106.6 | - 544.1 | .3469 | 1.770 | .60 | 99.68 | - 639.4 | .2380 | 1.527 |
| 84.56 | - 400.2 | .5054 | 2.392 | .65 | 80.23 | - 471.9 | .3502 | 2.060 |
| 73.74 | - 311.8 | .7357 | 3.028 | .70 | 72.58 | - 367.1 | .5183 | 2.622 |
| 72.87 | - 250.3 | 1.072 | 3.683 | .75 | 70.48 | - 293.3 | .7744 | 3.223 |
| 73.28 | - 204.4 | 1.554 | 4.335 | .80 | 71.62 | - 237.5 | 1.164 | 3.860 |
| 76.02 | - 168.4 | 2.227 | 4.934 | .85 | 75.22 | - 192.8 | 1.757 | 4.502 |
| 80.72 | - 141.4 | 3.117 | 5.376 | .90 | 81.00 | - 155.6 | 2.633 | 5.057 |
| 87.59 | - 115.5 | 4.193 | 5.531 | .95 | 89.08 | - 122.7 | 3.874 | 5.337 |
| 94.81 | - 94.30 | 5.302 | 5.274 | 1.00 | 98.94 | - 95.22 | 5.247 | 5.050 |
| 104.4 | - 77.20 | 6.193 | 4.579 | 1.05 | 111.8 | - 70.25 | 6.413 | 4.030 |
| 115.4 | - 63.08 | 6.672 | 3.648 | 1.10 | 127.6 | - 48.16 | 6.860 | 2.589 |
| 128.0 | - 52.23 | 6.696 | 2.732 | 1.15 | 147.1 | - 29.22 | 6.541 | 1.300 |
| 142.0 | - 44.54 | 6.412 | 2.011 | 1.20 | 171.2 | - 14.36 | 5.800 | .4866 |
| 157.4 | - 41.09 | 5.949 | 1.553 | 1.25 | 200.4 | - 5.427 | 4.986 | .1350 |
| 172.9 | - 41.89 | 5.462 | 1.323 | 1.30 | 234.0 | - 4.986 | 4.271 | .0910 |
| 188.1 | - 48.12 | 4.991 | 1.277 | 1.35 | 270.2 | - 17.55 | 3.686 | .2394 |
| 200.7 | - 60.73 | 4.565 | 1.382 | 1.40 | 302.2 | - 46.51 | 3.232 | .4975 |
| 207.4 | - 76.86 | 4.240 | 1.571 | 1.45 | 320.5 | - 91.53 | 2.885 | .8239 |
| 207.5 | - 95.54 | 3.977 | 1.831 | 1.50 | 315.5 | - 142.0 | 2.636 | 1.186 |
| 199.8 | - 113.3 | 3.788 | 2.148 | 1.55 | 287.5 | - 184.7 | 2.462 | 1.581 |
| 186.0 | - 126.1 | 3.684 | 2.497 | 1.60 | 247.0 | - 207.6 | 2.373 | 1.994 |
| 169.5 | - 132.7 | 3.657 | 2.864 | 1.65 | 206.3 | - 211.7 | 2.361 | 2.423 |
| 152.8 | - 133.1 | 3.722 | 3.242 | 1.70 | 172.5 | - 202.6 | 2.437 | 2.862 |
| 138.7 | - 128.9 | 3.869 | 3.595 | 1.75 | 147.8 | - 186.7 | 2.607 | 3.293 |

TABLE B1
(Continued)

| R | X | G · $10^3$ | B · $10^3$ | kb | R | X | G · $10^3$ | B · $10^3$ |
|---|---|---|---|---|---|---|---|---|
| 127.8 | -121.6 | 4.106 | 3.909 | 1.80 | 130.9 | -168.1 | 2.883 | 3.703 |
| 119.5 | -112.6 | 4.415 | 4.158 | 1.85 | 121.7 | -149.6 | 3.273 | 4.023 |
| 115.4 | -104.2 | 4.775 | 4.310 | 1.90 | 114.6 | -131.4 | 3.774 | 4.282 |
| 113.2 | - 95.82 | 5.146 | 4.356 | 1.95 | 114.8 | -115.1 | 4.345 | 4.357 |
| 113.4 | - 88.39 | 5.485 | 4.275 | 2.00 | 117.0 | -101.6 | 4.915 | 4.226 |
| 114.4 | - 82.18 | 5.766 | 4.142 | 2.05 | 121.7 | - 87.95 | 5.398 | 3.902 |
| 116.8 | - 77.20 | 5.958 | 3.938 | 2.10 | 128.8 | - 77.59 | 5.697 | 3.432 |
| 119.9 | - 73.74 | 6.050 | 3.720 | 2.15 | 137.4 | - 70.01 | 5.776 | 2.941 |
| 123.2 | - 71.84 | 6.056 | 3.531 | 2.20 | 148.0 | - 64.94 | 5.665 | 2.485 |
| 126.2 | - 71.22 | 6.001 | 3.385 | 2.25 | 159.1 | - 63.49 | 5.421 | 2.163 |
| 129.3 | - 71.99 | 5.903 | 3.286 | 2.30 | 170.4 | - 65.77 | 5.108 | 1.972 |
| 131.1 | - 73.82 | 5.792 | 3.263 | 2.35 | 180.0 | - 72.08 | 4.788 | 1.918 |
| 131.9 | - 76.29 | 5.681 | 3.286 | 2.40 | 186.8 | - 81.78 | 4.492 | 1.966 |
| 131.7 | - 79.05 | 5.581 | 3.349 | 2.45 | 190.6 | - 94.78 | 4.206 | 2.091 |
| 130.1 | - 81.71 | 5.512 | 3.461 | 2.50 | 187.8 | -106.4 | 4.031 | 2.284 |

## TABLE B2

**Impedance of Loop Antennae**

$\Omega = 10$; $2\pi b/a = 148.41$ $\qquad$ $\Omega = 11$; $2\pi b/a = 244.69$

| R | X | $G \cdot 10^3$ | $B \cdot 10^3$ | kb | R | X | $G \cdot 10^3$ | $B \cdot 10^3$ |
|---|---|---|---|---|---|---|---|---|
| .0051 | 62.59 | .0013 | -15.98 | .05 | .0047 | 72.24 | .0009 | -13.84 |
| .0410 | 128.0 | .0025 | - 7.812 | .10 | .0411 | 147.0 | .0019 | - 6.803 |
| .1577 | 203.7 | .0038 | - 4.908 | .15 | .1532 | 233.9 | .0028 | - 4.275 |
| .5936 | 297.7 | .0067 | - 3.360 | .20 | .5991 | 342.7 | .0051 | - 2.918 |
| 1.777 | 425.8 | .0098 | - 2.348 | .25 | 1.744 | 488.8 | .0073 | - 2.046 |
| 6.355 | 624.4 | .0163 | - 1.601 | .30 | 6.263 | 713.6 | .0123 | - 1.401 |
| 25.47 | 1003.1 | .0253 | - .9963 | .35 | 24.43 | 1136.6 | .0189 | - .8793 |
| 159.7 | 2063.4 | .0373 | - .4818 | .40 | 149.1 | 2294.6 | .0282 | - .4339 |
| 1679.4 | -3205.9 | .0571 | .0109 | .45 | 2263.2 | 5631.6 | .0430 | - .0107 |
| 468.8 | -2250.5 | .0887 | .4258 | .50 | 479.5 | -2768.6 | .0607 | .3505 |
| 156.6 | -1142.6 | .1177 | .8590 | .55 | 167.7 | -1360.8 | .0892 | .7238 |
| 100.7 | - 756.0 | .1731 | 1.300 | .60 | 10[illegible].0 | - 891.3 | .1316 | 1.106 |
| 80.95 | - 555.7 | .2567 | 1.762 | .65 | 84.42 | - 650.5 | .1962 | 1.512 |
| 73.25 | - 430.4 | .3842 | 2.258 | .70 | 75.87 | - 500.7 | .2959 | 1.952 |
| 71.23 | - 341.9 | .5841 | 2.803 | .75 | 73.50 | - 394.9 | .4556 | 2.448 |
| 72.60 | - 274.3 | .9018 | 3.407 | .80 | 74.78 | - 314.0 | .7175 | 3.013 |
| 76.57 | - 219.5 | 1.416 | 4.061 | .85 | 78.92 | - 248.3 | 1.162 | 3.657 |
| 82.99 | - 173.3 | 2.248 | 4.694 | .90 | 85.67 | - 192.1 | 1.937 | 4.342 |
| 91.97 | - 132.5 | 3.534 | 5.093 | .95 | 95.37 | - 141.5 | 3.274 | 4.859 |
| 103.7 | - 95.58 | 5.214 | 4.808 | 1.00 | 108.1 | - 95.57 | 5.192 | 4.591 |
| 119.2 | - 61.60 | 6.621 | 3.422 | 1.05 | 12[illegible].5 | - 51.61 | 6.816 | 2.804 |
| 138.9 | - 29.61 | 6.884 | 1.468 | 1.10 | 148.3 | - 8.771 | 6.720 | .3974 |
| 164.8 | - 12.50 | 6.067 | .0046 | 1.15 | 179.4 | 33.22 | 5.389 | - .9979 |
| 199.2 | 25.90 | 4.936 | - .6415 | 1.20 | 222.4 | 73.70 | 4.051 | - 1.342 |
| 244.4 | 45.67 | 3.954 | - .7390 | 1.25 | 282.8 | 109.6 | 3.074 | - 1.[illegible]92 |
| 302.2 | 53.12 | 3.210 | - .5642 | 1.30 | 367.3 | 132.4 | 2.410 | - .8682 |
| 371.0 | 37.06 | 2.669 | - .2666 | 1.35 | 480.3 | 121.7 | 1.956 | - .4955 |
| 438.4 | - 16.75 | 2.278 | .0870 | 1.40 | 604.5 | 50.75 | 1.643 | - .1379 |
| 475.9 | - 109.9 | 1.995 | .4607 | 1.45 | 677.1 | - 122.7 | 1.430 | .2592 |
| 455.1 | - 214.2 | 1.799 | .8465 | 1.50 | 627.6 | - 308.0 | 1.284 | .6302 |
| 384.9 | - 286.2 | 1.673 | 1.244 | 1.55 | 491.3 | - 412.2 | 1.195 | 1.002 |
| 302.8 | - 309.7 | 1.614 | 1.651 | 1.60 | 355.3 | - 425.4 | 1.156 | 1.385 |
| 234.4 | - 299.8 | 1.619 | 2.070 | 1.65 | 260.3 | - 393.3 | 1.170 | 1.768 |
| 185.2 | - 274.2 | 1.692 | 2.504 | 1.70 | 198.2 | - 346.3 | 1.245 | 2.175 |
| 153.5 | - 242.6 | 1.863 | 2.944 | 1.75 | 162.4 | - 295.3 | 1.430 | 2.600 |

TABLE B2
(Continued)

| R | X | G · 10^3 | B · 10^3 | kb | R | X | G · 10^3 | B · 10^3 |
|---|---|---|---|---|---|---|---|---|
| 133.7 | -211.3 | 2.139 | 3.380 | 1.80 | 138.1 | -253.9 | 1.653 | 3.040 |
| 122.6 | -181.7 | 2.551 | 3.731 | 1.85 | 126.3 | -213.3 | 2.055 | 3.471 |
| 118.1 | -154.5 | 3.122 | 4.086 | 1.90 | 122.1 | -176.4 | 2.653 | 3.832 |
| 118.5 | -129.8 | 3.836 | 4.202 | 1.95 | 124.0 | -142.6 | 3.471 | 3.994 |
| 123.2 | -107.5 | 4.610 | 4.019 | 2.00 | 130.5 | -111.5 | 4.429 | 3.786 |
| 131.6 | - 87.41 | 5.272 | 3.500 | 2.05 | 142.1 | - 82.63 | 5.259 | 3.058 |
| 143.9 | - 69.91 | 5.625 | 2.732 | 2.10 | 159.2 | - 56.28 | 5.583 | 1.974 |
| 159.8 | - 55.64 | 5.580 | 1.942 | 2.15 | 182.5 | - 33.00 | 5.307 | .9598 |
| 179.6 | - 45.83 | 5.228 | 1.335 | 2.20 | 212.8 | - 15.14 | 4.678 | .3328 |
| 202.5 | - 41.74 | 4.737 | .9764 | 2.25 | 250.7 | - 4.160 | 3.988 | .0662 |
| 227.6 | - 45.36 | 4.226 | .8424 | 2.30 | 295.8 | - 6.389 | 3.380 | .0730 |
| 251.3 | - 58.68 | 3.774 | .8812 | 2.35 | 342.5 | - 27.26 | 2.901 | .2309 |
| 269.6 | - 81.82 | 3.397 | 1.031 | 2.40 | 379.2 | - 70.53 | 2.535 | .4715 |
| 277.5 | -112.5 | 3.094 | 1.255 | 2.45 | 397.6 | -132.8 | 2.263 | .7559 |
| 271.3 | -144.3 | 2.873 | 1.528 | 2.50 | 381.6 | -196.0 | 2.074 | 1.065 |

## TABLE B3

**Impedance of Loop Antennas**

$\Omega = .12; \; 2\pi b/a = 403.43$

| kb | R | X | $G \cdot 10^3$ | $B \cdot 10^3$ |
|---|---|---|---|---|
| .05 | .0053 | 81.46 | .0008 | -12.28 |
| .10 | .0419 | 167.0 | .0015 | - 5.986 |
| .15 | .1548 | 265.3 | .0022 | - 3.769 |
| .20 | .5967 | 386.2 | .0040 | - 2.589 |
| .25 | 1.721 | 549.5 | .0057 | - 1.820 |
| .30 | 6.042 | 797.4 | .0095 | - 1.254 |
| .35 | 23.50 | 1255.6 | .0149 | - .7962 |
| .40 | 131.5 | 2446.6 | .0219 | - .4076 |
| .45 | 1465.2 | 14,782.6 | .0337 | - .0340 |
| .50 | 588.8 | -3477.0 | .0474 | .2799 |
| .55 | 185.6 | -1617.7 | .0700 | .6102 |
| .60 | 106.1 | -1011.2 | .1033 | .9842 |
| .65 | 89.32 | - 754.6 | .1547 | 1.307 |
| .70 | 79.48 | - 576.3 | .2348 | 1.703 |
| .75 | 76.52 | - 451.4 | .3650 | 2.153 |
| .80 | 77.55 | - 356.1 | .5839 | 2.631 |
| .85 | 81.63 | - 276.4 | .9698 | 3.308 |
| .90 | 88.50 | - 211.6 | 1.682 | 4.022 |
| .95 | 98.43 | - 151.5 | 3.017 | 4.642 |
| 1.00 | 112.0 | - 95.32 | 5.178 | 4.406 |
| 1.05 | 130.5 | - 41.05 | 6.974 | - 2.194 |
| 1.10 | 155.3 | 13.30 | 6.393 | - .5478 |
| 1.15 | 190.7 | 67.58 | 4.660 | - 1.652 |
| 1.20 | 239.[illegible] | 125.4 | 3.278 | - 1.718 |
| 1.25 | 312.5 | 181.6 | 2.392 | - 1.390 |
| 1.30 | 422.0 | 227.9 | 1.834 | - .9905 |
| 1.35 | 584.2 | 236.5 | 1.471 | - 5954 |
| 1.40 | 788.1 | 142.3 | 1.229 | - .2218 |
| 1.45 | 926.1 | - 116.0 | 1.063 | .1332 |
| 1.50 | 837.1 | - 419.6 | .9546 | .4785 |
| 1.55 | 610.9 | - 559.7 | .8899 | .8153 |
| 1.60 | 414.5 | - 554.2 | .8654 | 1.157 |
| 1.65 | 287.7 | - 493.0 | .8829 | 1.513 |
| 1.70 | 212.5 | - 422.5 | .9503 | 1.889 |
| 1.75 | 168.8 | - 356.6 | 1.085 | 2.291 |

TABLE B3
(Continued)

| kb | R | X | $G \cdot 10^3$ | $B \cdot 10^3$ |
|---|---|---|---|---|
| 1.80 | 144.0 | -297.6 | 1.317 | 2.722 |
| 1.85 | 131.3 | -245.3 | 1.696 | 3.168 |
| 1.90 | 123.8 | -193.8 | 2.295 | 3.594 |
| 1.95 | 129.7 | -154.6 | 3.185 | 3.796 |
| 2.00 | 137.6 | -113.9 | 4.314 | 3.571 |
| 2.05 | 151.7 | - 75.20 | 5.292 | 2.623 |
| 2.10 | 172.7 | - 38.28 | 5.518 | 1.223 |
| 2.15 | 195.8 | - 36.02 | 4.941 | .0909 |
| 2.20 | 243.0 | 26.49 | 4.067 | - .4433 |
| 2.25 | 257.2 | 48.43 | 3.277 | - .5340 |
| 2.30 | 367.0 | 53.74 | 2.668 | - .3907 |
| 2.35 | 447.1 | 28.71 | 2.227 | - .1430 |
| 2.40 | 520.0 | - 39.84 | 1.912 | .1465 |
| 2.45 | 552.7 | -148.0 | 1.688 | .4522 |
| 2.50 | 521.0 | -259.2 | 1.538 | .7641 |

## TABLE B4

**Input Impedance for ka Constant**

| | a = 3/16 in at λ = 100 cm | | a = 1/4 in at λ = 100 cm | | a = 5/16 in at λ = 100 cm | | |
|---|---|---|---|---|---|---|---|
| Kb | R | X | R | X | R | X | Kb |
| .05 | | | | | | | .05 |
| .10 | | | | | | | .10 |
| .15 | | 149 | | 133 | | | .15 |
| .20 | | 244 | | 217 | | 196 | .20 |
| .25 | 2 | 385 | 2 | 357 | | 313 | .25 |
| .30 | 7 | 625 | 6 | 526 | 6 | 500 | .30 |
| .35 | 25 | 999 | 27 | 908 | 23 | 801 | .35 |
| .40 | 172 | 2209 | 167 | 1986 | 169 | 1871 | .40 |
| .45 | 19992 | -3998 | 15202 | -5153 | 13535 | -3981 | .45 |
| .50 | 376 | -2320 | 408 | -2251 | 410 | -2094 | .50 |
| .55 | 170 | -1348 | 152 | -1200 | 129 | - 963 | .55 |
| .60 | 104 | - 905 | 103 | - 820 | 103 | - 767 | .60 |
| .65 | 85 | - 670 | 83 | - 613 | 81 | - 570 | .65 |
| .70 | 77 | - 525 | 75 | - 486 | 74 | - 455 | .70 |
| .75 | 75 | - 422 | 73 | - 391 | 72 | - 367 | .75 |
| .80 | 76 | - 342 | 75 | - 317 | 74 | - 298 | .80 |
| .85 | 81 | - 270 | 80 | - 255 | 78 | - 242 | .85 |
| .90 | 89 | - 210 | 87 | - 197 | 86 | - 188 | .90 |
| .95 | 98 | - 155 | 97 | - 150 | 95 | - 142 | .95 |
| 1.00 | 112 | - 102 | 110 | - 95 | 108 | - 95 | 1.00 |
| 1.05 | 132 | - 39 | 129 | - 45 | 126 | - 50 | 1.05 |
| 1.10 | 157 | 20 | 154 | 6 | 151 | - 3 | 1.10 |
| 1.15 | 195 Ω | 81 | 189 | 60 | 184 | 45 | 1.15 |
| 1.20 | 245 (12.447) | 152 | 238 | 118 | 231 | 95 | 1.20 |
| 1.25 | | | 311 | 178 | 299 | 145 | 1.25 |
| 1.30 | | | 426 | 229 | 402 | 186 | 1.30 |
| 1.35 | | | 589 | 244 | 547 | 191 | 1.35 |
| 1.40 | | | 836 | 167 | 739 | 118 | 1.40 |
| 1.45 | | | 988 | - 108 | 853 | - 119 | 1.45 |
| 1.50 | | | 918 | - 448 | 800 | - 400 | 1.50 |
| 1.55 | | | 689 Ω | - 613 | 600 | - 547 | 1.55 |
| 1.60 | | | 447 (12.446) | - 623 | 418 | - 553 | 1.60 |
| 1.65 | | | | | 289 | - 497 | 1.65 |
| 1.70 | | | | | 215 | - 434 | 1.70 |
| 1.75 | | | | | 170 | - 365 | 1.75 |
| 1.80 | | | | | 146 | - 309 | 1.80 |
| 1.85 | | | | | 134 | - 252 | 1.85 |
| 1.90 | | | | | 126 | - 200 | 1.90 |
| 1.95 | | | | | 132 | - 160 | 1.95 |
| 2.00 | | | | | 141 Ω | - 113 | 2.00 |
| 2.05 | | | | | 157 (12.494) | - 72 | 2.05 |

TABLE B5

Input Impedance of Loop Antenna

for ka Constant

| Kb | R (a = 3/8 in at λ = 100 cm) | X (a = 3/8 in at λ = 100 cm) | R (a = 1/2 in at λ = 100 cm) | X (a = 1/2 in at λ = 100 cm) | R (a = 3/4 in at λ = 100 cm) | X (a = 3/4 in at λ = 100 cm) | Kb |
|---|---|---|---|---|---|---|---|
| .05 | | | | | | | .05 |
| .10 | | | | | | | .10 |
| .15 | | | | | | | .15 |
| .20 | | | | | | | .20 |
| .25 | 2 | 278 | | | | | .25 |
| .30 | 6 | 454 | 6 | 416 | | | .30 |
| .35 | 24 | 768 | 22 | 666 | | | .35 |
| .40 | 171 | 1738 | 146 | 1456 | | | .40 |
| .45 | 1216 | -3160 | 1052 | | 6772 | 2646 | .45 |
| .50 | 400 | -1959 | 431 | -1821 | 513 | -1660 | .50 |
| .55 | 153 | -1053 | 157 | - 954 | 166 | - 837 | .55 |
| .60 | 100 | - 716 | 95 | - 653 | 103 | - 570 | .60 |
| .65 | 81 | - 542 | 80 | - 495 | 82 | - 435 | .65 |
| .70 | 73 | - 430 | 73 | - 390 | 73 | - 345 | .70 |
| .75 | 72 | - 348 | 71 | - 318 | 71 | - 280 | .75 |
| .80 | 73 | - 283 | 72 | - 261 | 72 | - 232 | .80 |
| .85 | 78 | - 230 | 76 | - 215 | 76 | - 192 | .85 |
| .90 | 84 | - 182 | 83 | - 170 | 82 | - 158 | .90 |
| .95 | 94 | - 138 | 92 | - 133 | 90 | - 125 | .95 |
| 1.00 | 106 | - 95 | 104 | - 95 | 101 | - 95 | 1.00 |
| 1.05 | 124 | - 53 | 122 | - 60 | 115 | - 67 | 1.05 |
| 1.10 | 148 | - 12 | 142 | - 23 | 134 | - 40 | 1.10 |
| 1.15 | 179 | 33 | 171 | 13 | 158 | - 11 | 1.15 |
| 1.20 | 224 | 77 | 212 | 48 | 192 | 13 | 1.20 |
| 1.25 | 287 | 120 | 268 | 83 | 234 | 33 | 1.25 |
| 1.30 | 379 | 150 | 343 | 98 | 291 | 41 | 1.30 |
| 1.35 | 511 | 151 | 448 | 94 | 361 | 30 | 1.35 |
| 1.40 | 661 | 80 | 560 | 35 | 434 | - 15 | 1.40 |
| 1.45 | 774 | - 123 | 632 | - 116 | 487 | - 112 | 1.45 |
| 1.50 | 719 | - 353 | 617 | - 295 | 476 | - 264 | 1.50 |
| 1.55 | 552 | - 497 | 492 | - 410 | 402 | - 307 | 1.55 |
| 1.60 | 389 | - 508 | 356 | - 428 | 316 | - 337 | 1.60 |
| 1.65 | 280 | - 460 | 254 | - 404 | 243 | - 327 | 1.65 |
| 1.70 | 210 | - 405 | 202 | - 360 | 191 | - 300 | 1.70 |
| 1.75 | 168 | - 343 | 164 | - 310 | 159 | - 265 | 1.75 |

TABLE B5

(Continued)

| | a = 3/8 in at λ = 100 cm | | a = 1/2 in at λ = 100 cm | | a = 3/4 in at λ = 100 cm | | |
|---|---|---|---|---|---|---|---|
| Kb | R | X | R | X | R | X | Kb |
| 1.80 | 143 | -291 | 140 | -266 | 136 | -232 | 1.80 |
| 1.85 | 133 | -241 | 128 | -225 | 125 | -200 | 1.85 |
| 1.90 | 124 | -193 | 122 | -182 | 118 | -168 | 1.90 |
| 1.95 | 130 | -155 | 126 | -150 | 122 | -138 | 1.95 |
| 2.00 | 138 | -112 | 134 | -111 | 128 | -110 | 2.00 |
| 2.05 | 153 | - 75 | 147 | - 79 | 139 | - 85 | 2.05 |
| 2.10 | 181 | - 35 | 170 | - 45 | 156 | - 60 | 2.10 |
| 2.15 | 211 | 5 | 193 | - 15 | 179 | - 40 | 2.15 |
| 2.20 | 250 | 40 | 235 | 15 | 209 | - 18 | 2.20 |
| 2.25 | 310 | 65 | 286 | 34 | 248 | - 5 | 2.25 |
| 2.30 | 395 | 80 | 353 | 39 | 294 | - 8 | 2.30 |
| 2.35 | 489 | 53 | 427 | 16 | 343 | - 27 | 2.35 |
| 2.40 | 598 | - 25 | 499 | - 42 | 387 | - 70 | 2.40 |
| 2.45 | 653 Ω (12.486) | -174 | 541 | -148 | 409 | -136 | 2.45 |
| 2.50 | | | 517 Ω (11.951) | -262 | 398 Ω (11.141) | -201 | 2.50 |

## Appendix C

### Graphs to Facilitate Evaluation of the Current Distribution on a Loop Antenna

The current distribution on a loop antenna is given explicitly by equation (20)

$$I(\phi) = \frac{V}{j\pi \zeta_o} \left\{ \frac{1}{a_o} + 2 \sum_{1}^{4} \frac{\cos n\phi}{a_n} - \frac{2\pi}{\ln \frac{n_o}{4.5}} \left[ \left(\frac{kb}{4.5}\right) J_1(\phi) + \left(\frac{kb}{4.5}\right)^3 J_2(\phi) \right] \right\}$$

$$kb \leqq 2.5$$

where

V is the voltage driving the antenna

$\zeta_o$ = 120 ohms

a = radius of antenna wire

b = radius of antenna

$k = \omega/c = 2\pi/\lambda$

$\ln(n_o/4.5) = \frac{\Omega}{2} - 3.226$

$\Omega = 2 \ln \frac{2\pi b}{a}$

To facilitate evaluation of this formula, the succeeding pages contain the following graphs:

Figure C1: $\mathrm{Re}(1/a_o)$; $\Omega$ = 8, 9, 10, 11, 12, $kb \leqq 2.5$

Figure C2: $\mathrm{Im}(1/a_o)$; $\Omega$ = 8, 9, 10, 11, 12, $kb \leqq 2.5$

Figure C3: $\mathrm{Re}(1/a_1)$; $\Omega$ = 8, 9, 10, 11, 12, $kb \leqq 2.5$

Figure C4: $\mathrm{Im}(1/a_1)$; $\Omega$ = 8, 9, 10, 11, 12, $kb \leqq 2.5$

Figure C5: $\mathrm{Re}(1/a_2)$; $\Omega$ = 8, 9, 10, 11, 12, $kb \leqq 2.5$

Figure C6: $\mathrm{Im}(1/a_2)$; $\Omega$ = 8, 9, 10, 11, 12, $kb \leqq 2.5$

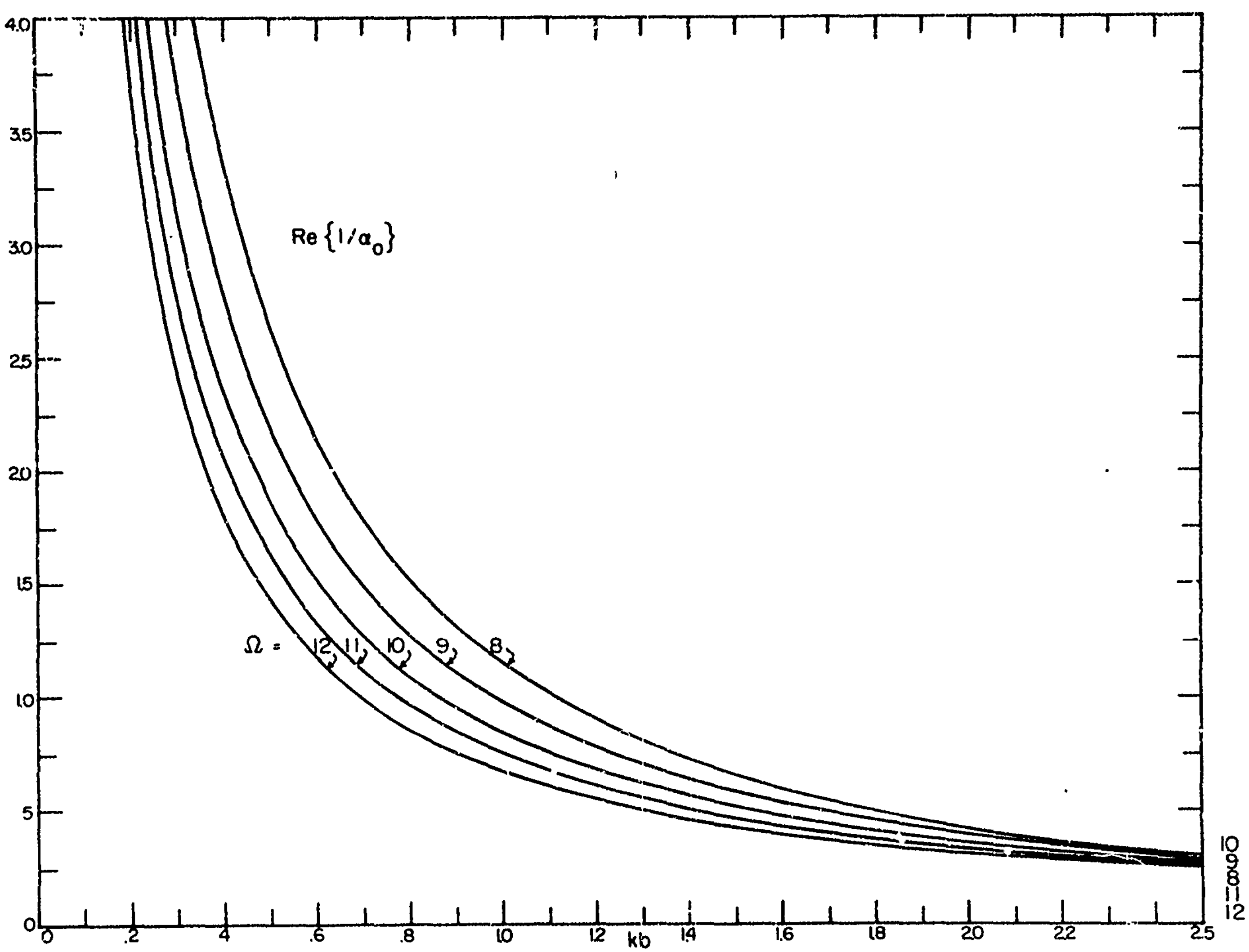
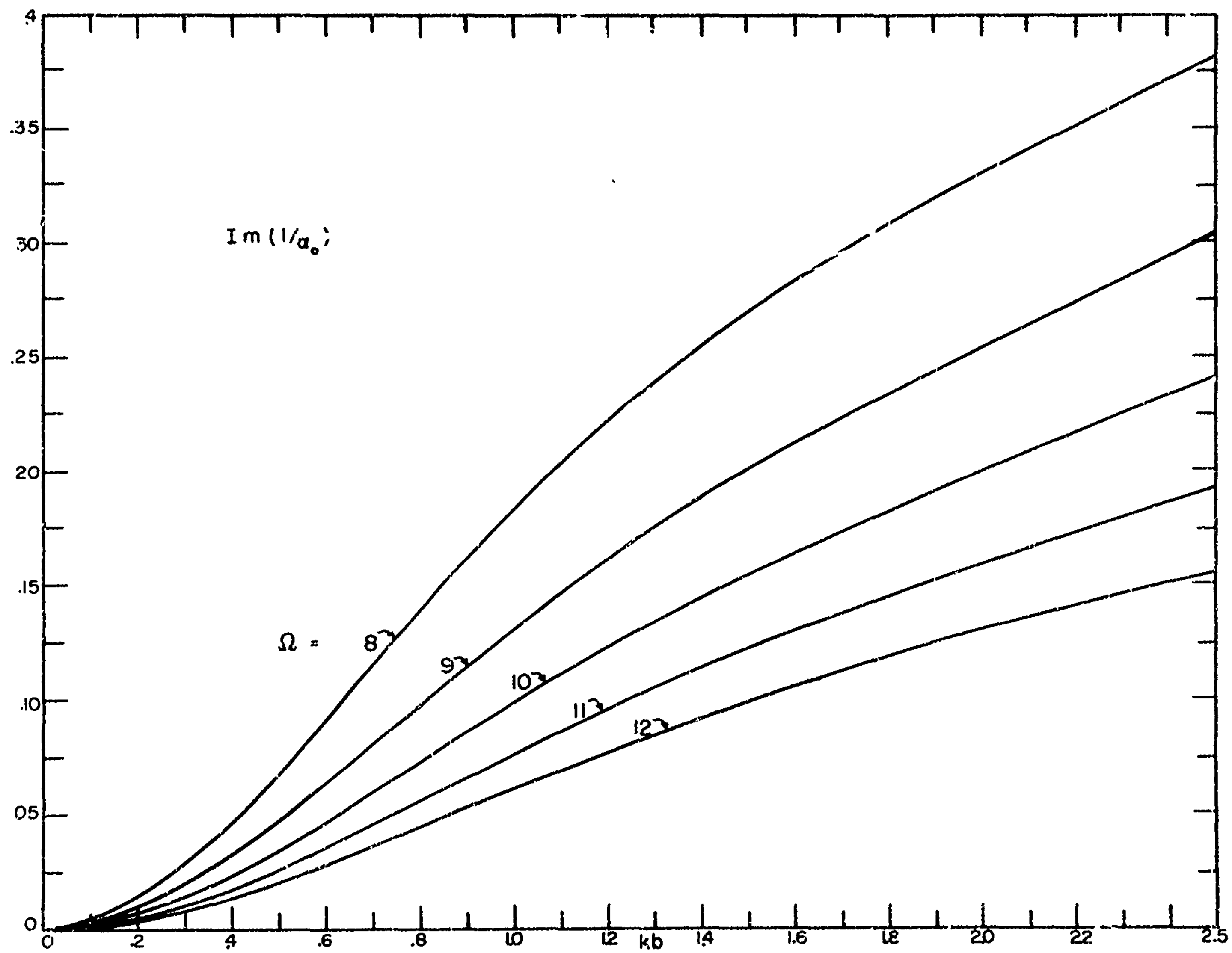

Re $\{1/\alpha_0\}$
$\Omega$ = 12 11 10 9 8
kb
10
9
8
11
12
4.0
3.5
3.0
2.5
2.0
1.5
1.0
.5
0
0
.2
.4
.6
.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
2.5

FIG. c1



Im $(1/a_0)$
$\Omega$ =
8
9
10
11
12
.4
.35
.30
.25
.20
.15
.10
.05
0
.2
.4
.6
.8
1.0
1.2
kb
1.4
1.6
1.8
2.0
2.2
2.5

FIG. c 2

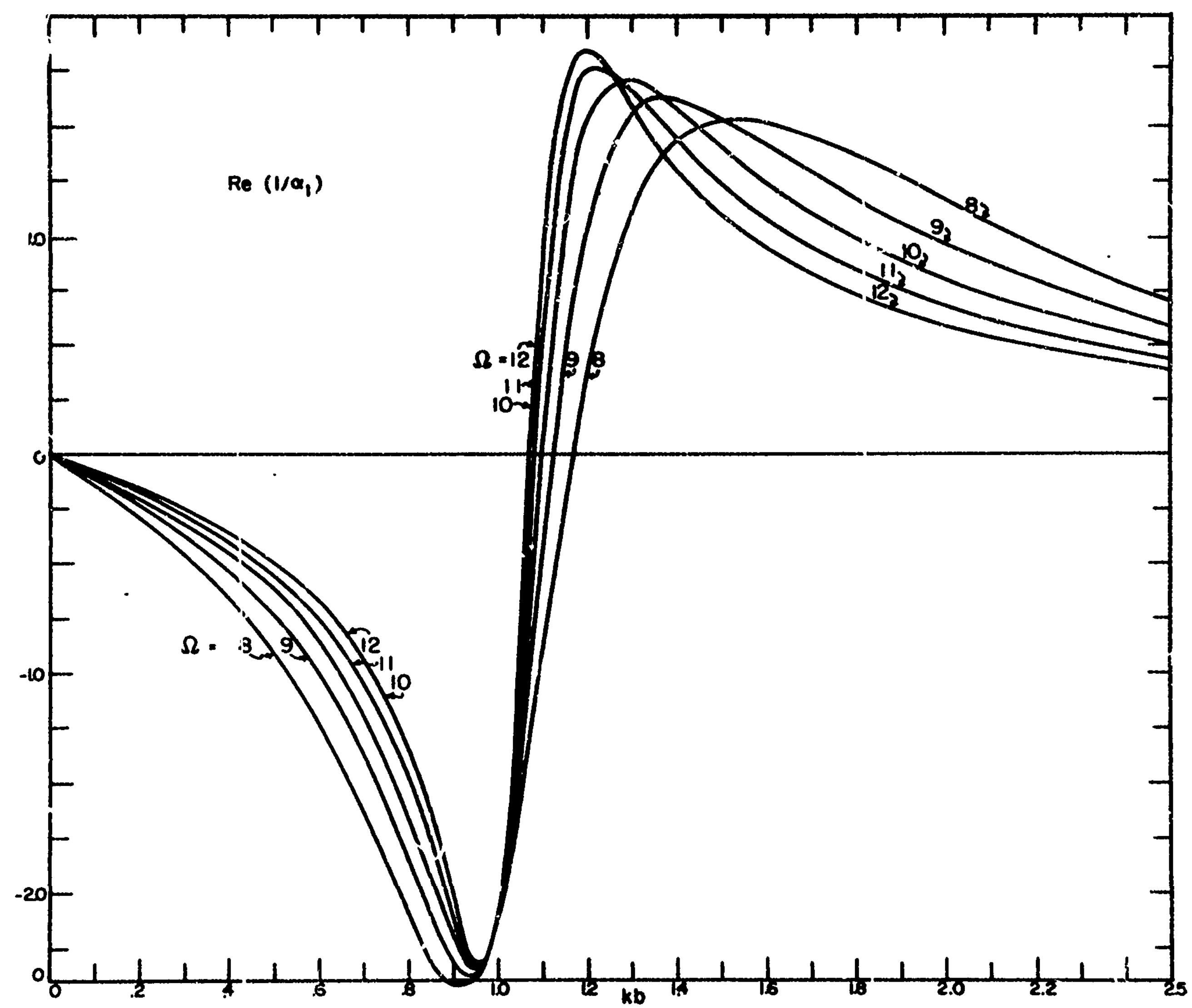



FIG. c3

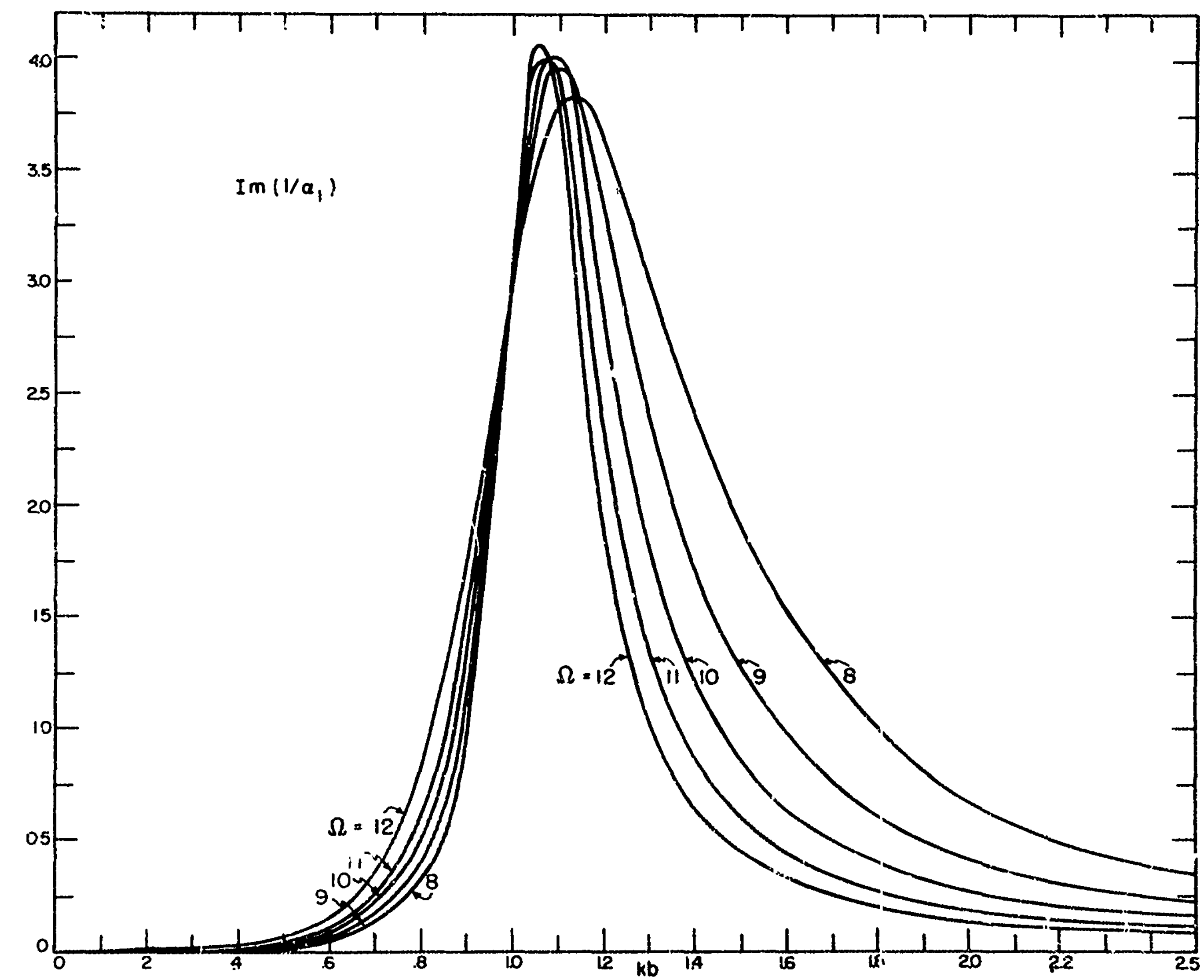
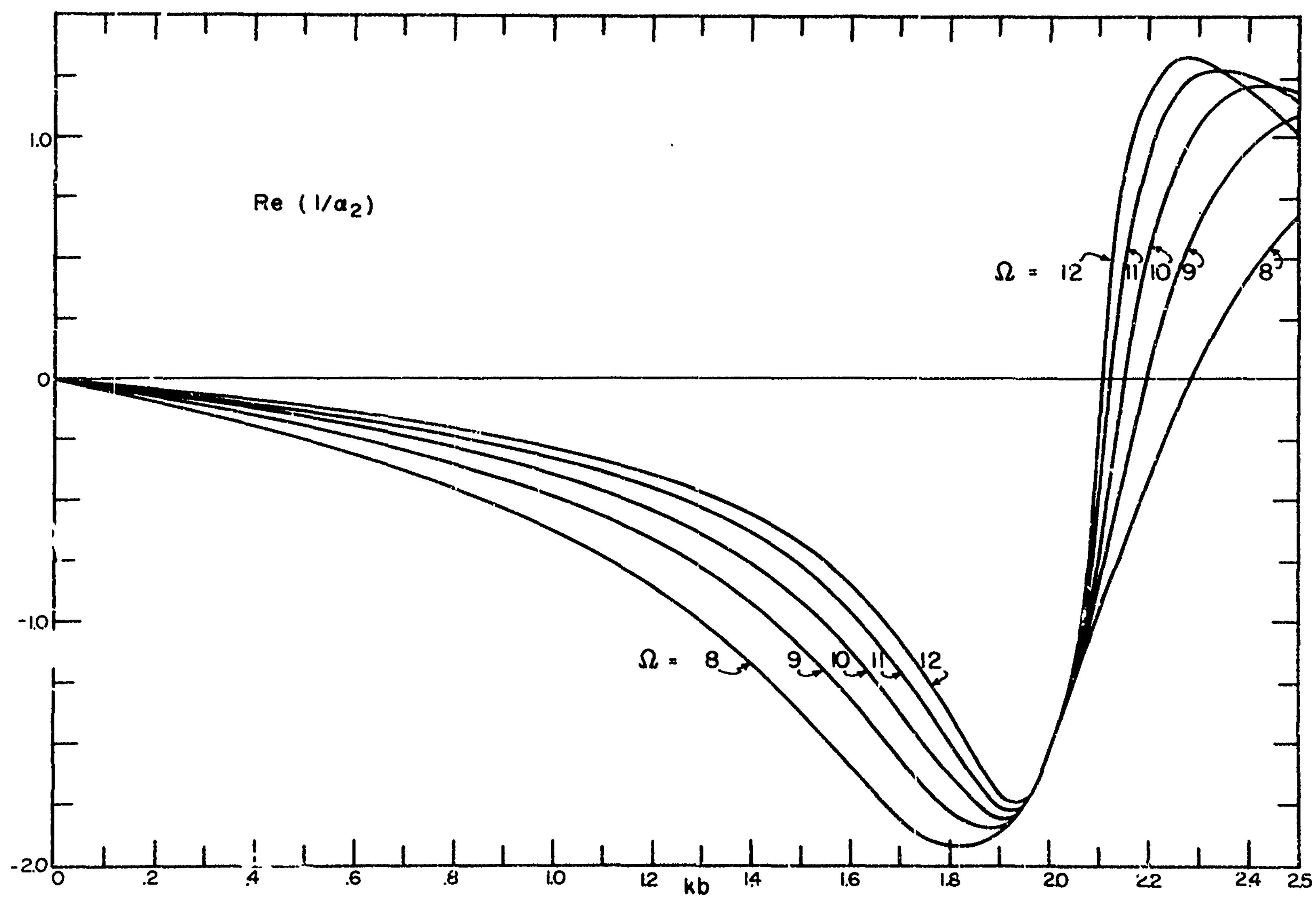
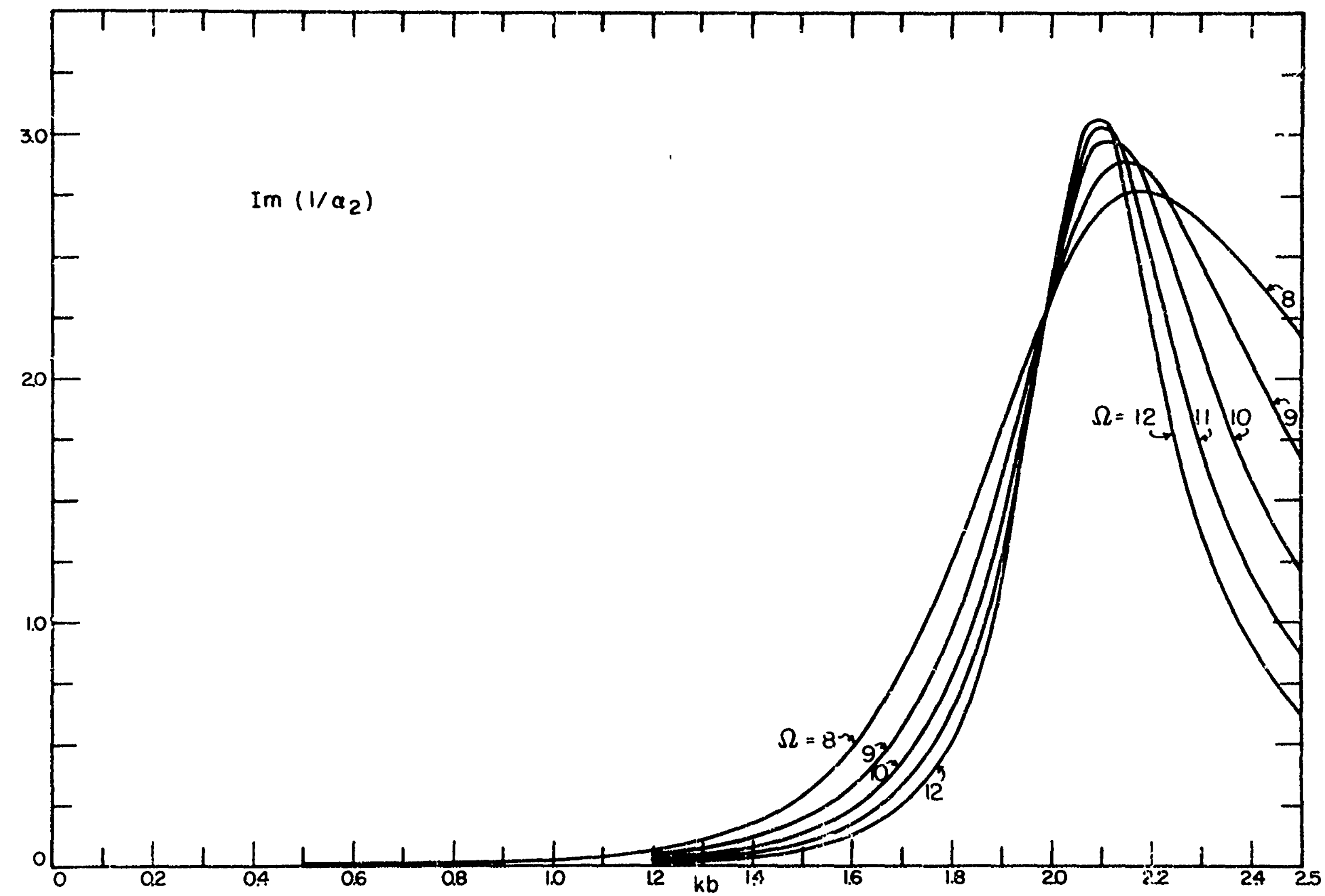



FIG. c4



Re ( 1/α2 )
Ω = 12 11 10 9 8
Ω = 8 9 10 11 12
1.0
0
-1.0
-2.0
0
.2
.4
.6
.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
2.4
2.5
kb

FIG. c5



Im (1/a2)
3.0
2.0
1.0
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
2.4
2.5
kb
Ω = 8
9
10
12
Ω = 12
11
10
9
8


FIG. c6

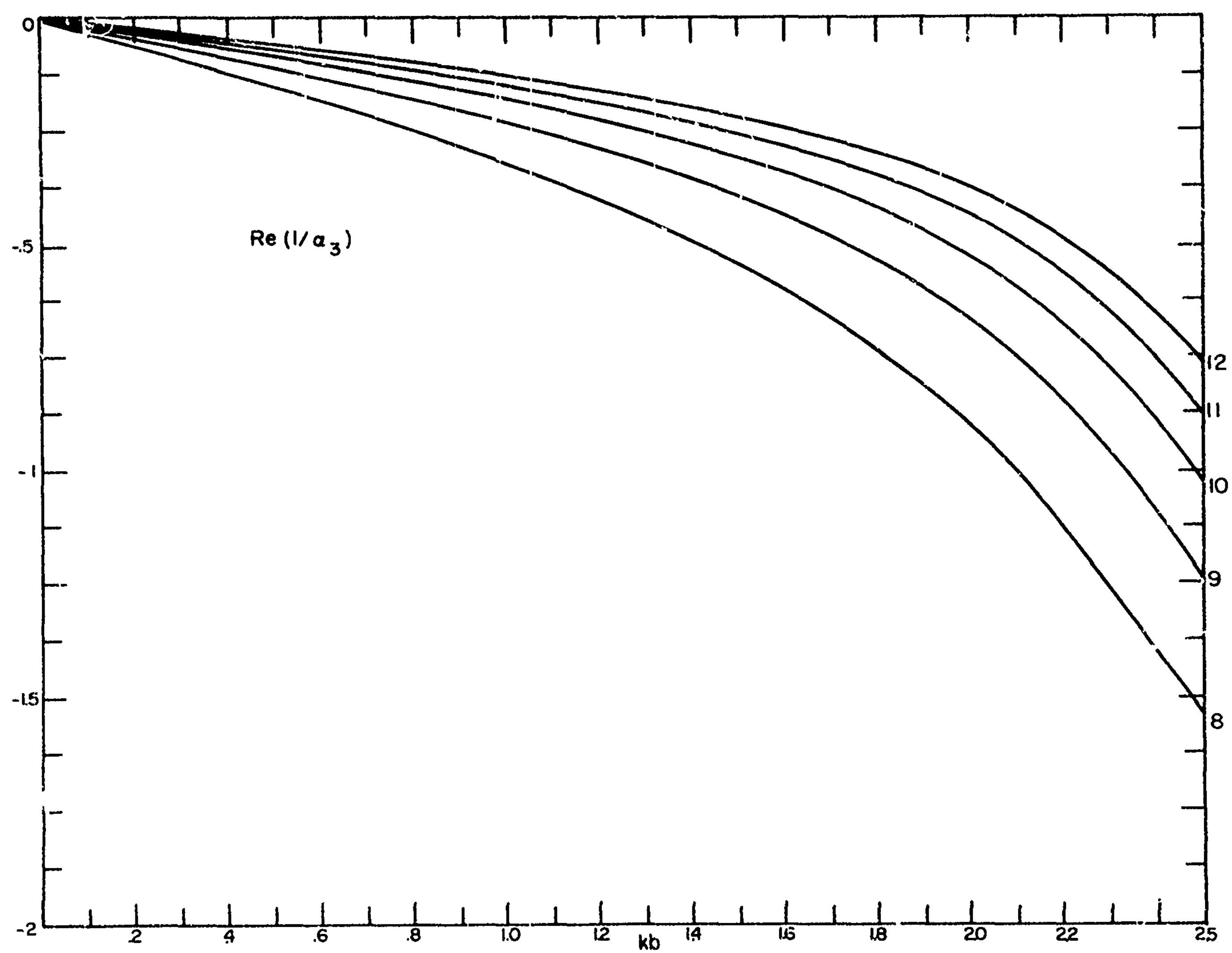


FIG. c7

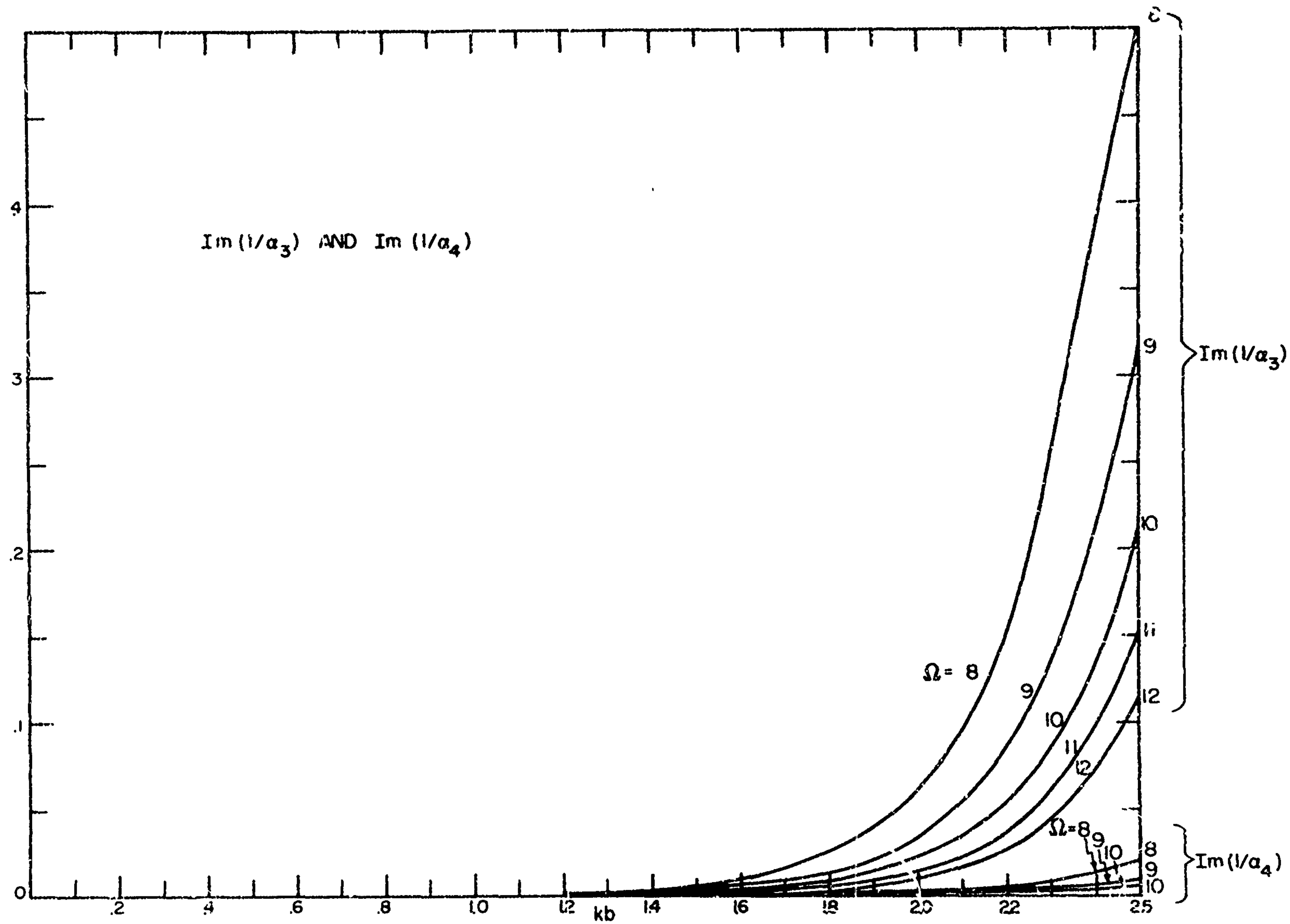



FIG. c8

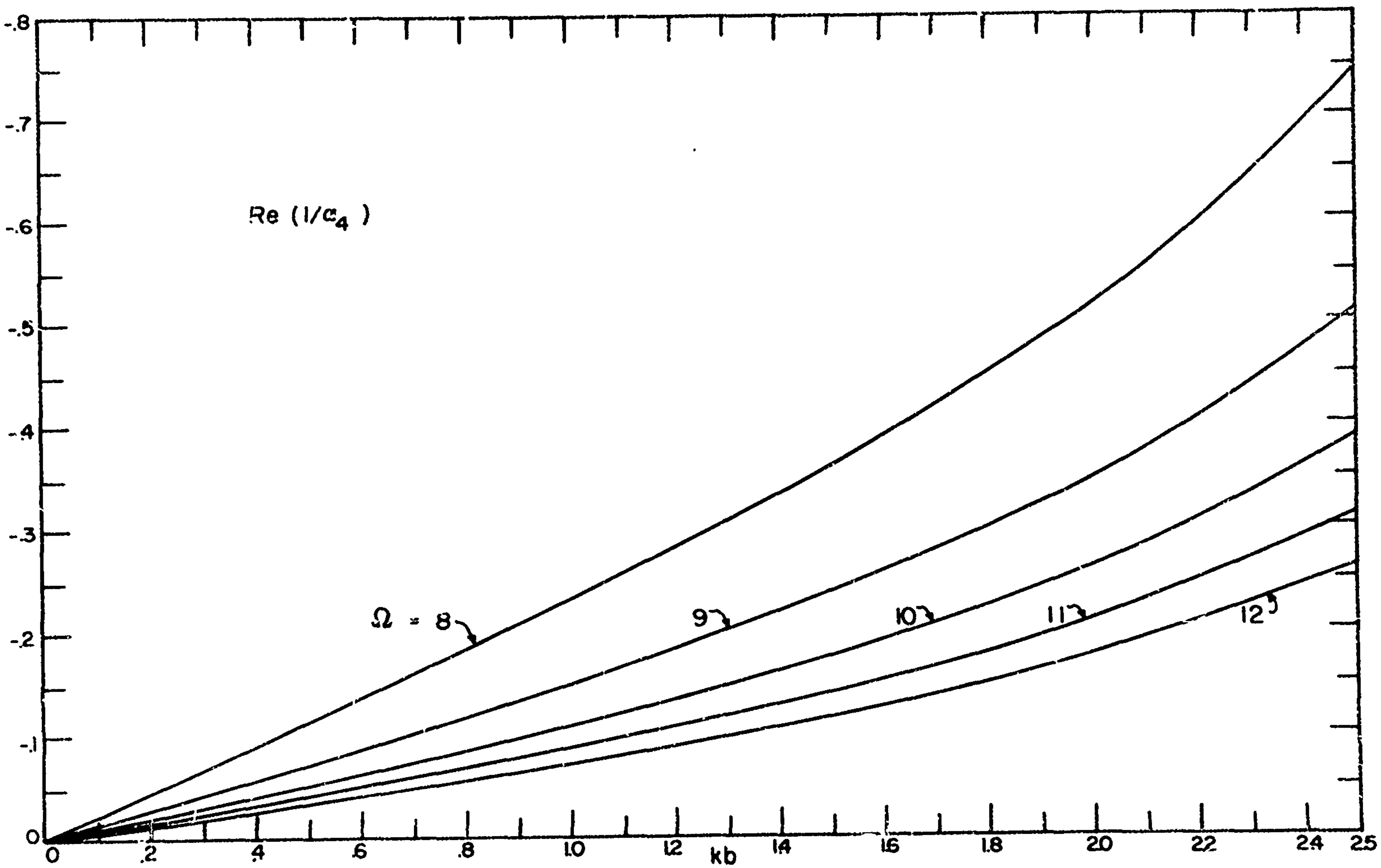
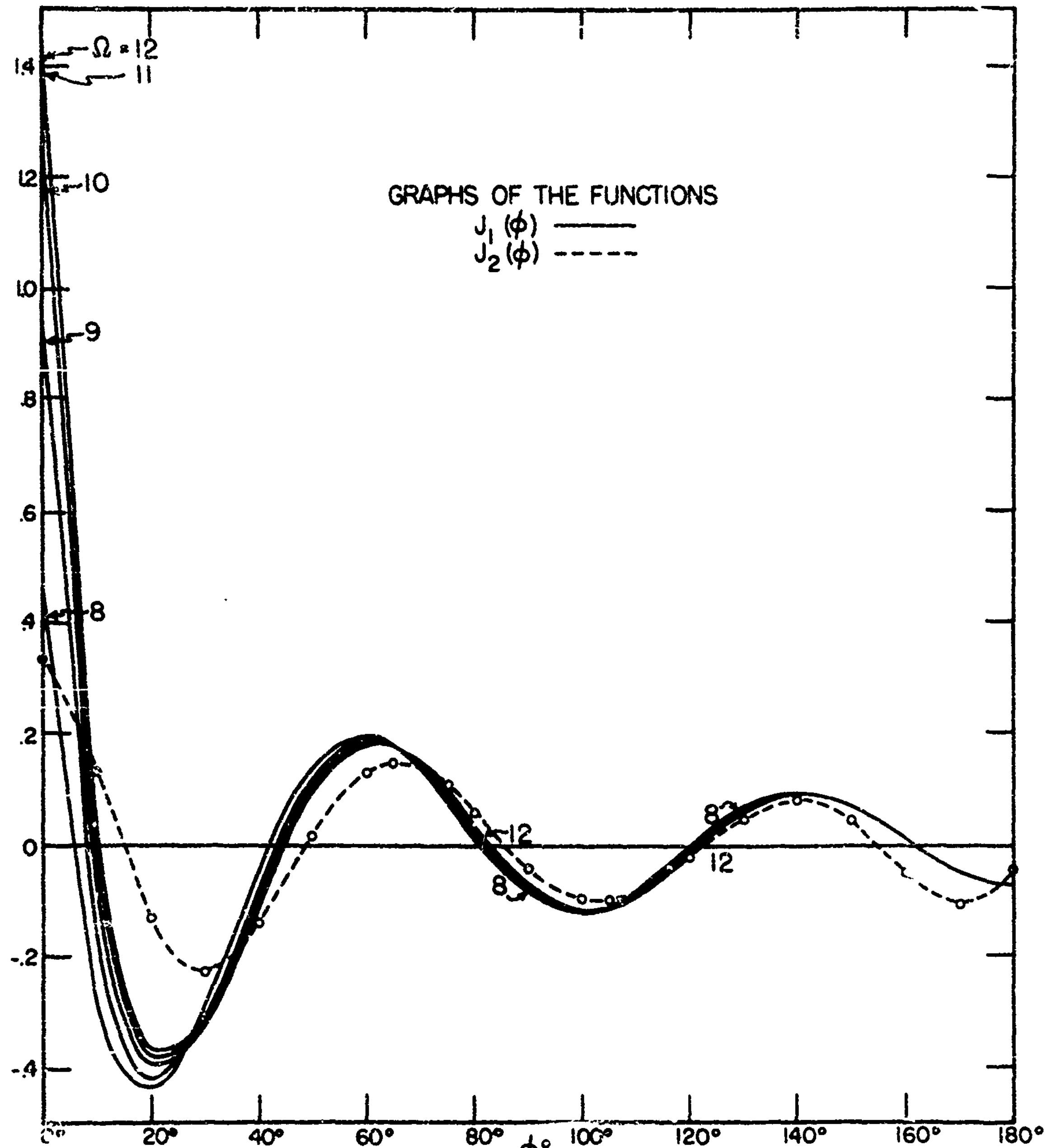



FIG. c9



GRAPHS OF THE FUNCTIONS
$J_1(\phi)$
$J_2(\phi)$
$\Omega = 12$
11
10
9
8
12
8
8
12
1.4
1.2
1.0
.8
.6
.4
.2
0
-.2
-.4
20°
40°
60°
80°
$\phi°$
100°
120°
140°
160°
180°


FIG. c10

DISTRIBUTION LIST

Technical Reports

| | |
|---|---|
| | Chief of Naval Research<br>Department of the Navy<br>Washington 25, C. C. |
| 2 | 427 |
| 1 | 460 |
| 1 | 421 |
| 6 | Director (Code 5250)<br>Naval Research Laboratory<br>Washington 25, D. C. |
| 2 | Commanding Officer<br>Office of Naval Research Branch Office<br>150 Causeway Street<br>Boston, Massachusetts |
| 1 | Commanding Officer<br>Officer of Naval Research Branch Office<br>1000 Geary Street<br>San Francisco 9, California |
| 1 | Commanding Officer<br>Office of Naval Research Branch Office<br>1030 E. Green Street<br>Pasadena, California |
| 1 | Commanding Officer<br>Office of Naval Research Branch Office<br>The John Crerar Library Building<br>86 East Randolph Street<br>Chicago 1, Illinois |
| 1 | Commanding Officer<br>Office of Naval Research Branch Office<br>346 Broadway<br>New York 13, New York |
| 3 | Officer-in-Charge<br>Office of Naval Research<br>Navy No. 100<br>Fleet Post Office<br>New York, New York |
| | Chief, Bureau of Ordnance<br>Navy Department<br>Washington 25, D. C. |
| 1 | Re4-1 |
| 1 | AD-3 |

Technical Reports

| | |
|---|---|
| | Chief, Bureau of Aeronautics<br>Navy Department<br>Washington 25, D. C. |
| 1 | EL-1 |
| 1 | EL-4 |
| 2 | Chief, Bureau of Ships (810)<br>Navy Department<br>Washington 25, D. C. |
| 1 | Chief of Naval Operations<br>Navy Department<br>Washington 25, D. C. |
| 1 | Op-413 |
| 1 | Op-20 |
| 1 | Op-32 |
| 1 | Director<br>Naval Ordnance Laboratory<br>White Oak, Maryland |
| 2 | Commander<br>U. S. Naval Electronics Laboratory<br>San Diego, California |
| 1 | Commander(AAEL)<br>Naval Air Development Center<br>Johnsville, Pennsylvania |
| 1 | Librarian<br>U. S. Naval Post Graduate School<br>Monterey, California |
| 50 | Transportation Officer<br>Building 151<br>Squier Signal Laboratory<br>Fort Monmouth, New Jersey<br>Attn: Director of Research |
| | Commanding General<br>Air Research and Development Command<br>Post Office Box 1395<br>Baltimore 3, Maryland |
| 3 | RDTRRP |
| 2 | RDDDE |
| 1 | Commanding General(WCRR)<br>Wright Air Development Center<br>Wright-Patterson Air Force Base, Ohio |

Technical Reports

| | |
|---|---|
| | Commanding General<br>Wright Air Development Center<br>Wright-Patterson Air Force Base, Ohio |
| 1 | WCRRH |
| 2 | WCLR |
| 1 | WCLRR |
| 2 | Technical Library |
| 2 | Commander<br>Wright Air Development Center<br>Attn: WCREO<br>Wright-Patterson Air Force Base, Ohio |
| | Commanding General<br>Rome Air Development Center<br>Griffiss Air Force Base<br>Rome, New York |
| 1 | RCREC-4C |
| 1 | RCR-1 |
| 2 | RCRW |
| | Commanding General<br>Air Force Cambridge Research Center<br>230 Albany Street<br>Cambridge 39, Massachusetts |
| 6 | CRR |
| 1 | Technical Library |
| 2 | Director<br>Air University Library<br>Maxwell Air Force Base, Alabama |
| 1 | Commander<br>Patrick Air Force Base<br>Cocoa, Florida |
| 1 | Chief, European Office<br>Air Research and Development Command<br>Shell Building<br>60 Rue Ravenstein<br>Brussels, Belgium |
| 1 | U. S. Coast Guard (EEE)<br>1300 E Street, N. W.<br>Washington, D. C. |
| 1 | Assistant Secretary of Defense<br>(Research and Development<br>Research and Development Board<br>Department of Defense<br>Washington 25, D. C. |

Technical Reports

| | |
|---|---|
| 5 | Armed Services Technical Information Agency<br>Document Service Center<br>Knott Building<br>Dayton 2, Ohio |
| 1 | Office of Technical Services<br>Department of Commerce<br>Washington 25, D. C. |
| 1 | Commanding Officer and Director<br>U. S. Underwater Sound Laboratory<br>New London, Connecticut |
| 1 | Federal Telecommunications Laboratories, Inc.<br>Technical Library<br>500 Washington Avenue<br>Nutley, New Jersey |
| 1 | Librarian<br>Radio Corporation of America<br>RCA Laboratories<br>Princeton, New Jersey |
| 1 | Sperry Gyroscope Company<br>Engineering Librarian<br>Great Neck, L. I., New York |
| 1 | Watson Laboratories<br>Library<br>Red Bank, New Jersey |
| 1 | Professor E. Weber<br>Polytechnic Institute of Brooklyn<br>9[illegible] Livingston Street<br>Brooklyn 2, New York |
| 1 | University of California<br>Department of Electrical Engineering<br>Berkeley, California |
| 1 | Dr. E. T. Booth<br>Hudson Laboratories<br>145 Palisade Street<br>Dobbs Ferry, New York |
| 1 | Cornell University<br>Department of Electrical Engineering<br>Ithaca, New York |
| 1 | University of Illinois<br>Department of Electrical Engineering<br>Urbana, Illinois |

Technical Reports

| | |
|---|---|
| 1 | Johns Hopkins University<br>Applied Physics Laboratory<br>Silver Spring, Maryland |
| 1 | Professor A. von Hippel<br>Massachusetts Institute of Technology<br>Research Laboratory for Insulation Research<br>Cambridge, Massachusetts |
| 1 | Director<br>Lincoln Laboratory<br>Massachusetts Institute of Technology<br>Cambridge 39, Massachusetts |
| 1 | Mr. A. D. Bedrosian<br>Room 22A-209<br>Signal Corps Liaison Office<br>Massachusetts Institute of Technology<br>Cambridge, Massachusetts |
| 1 | Mr. Hewitt<br>Massachusetts Institute of Technology<br>Document Room<br>Research Laboratory of Electronics<br>Cambridge, Massachusetts |
| 1 | Stanford University<br>Electronics Research Laboratory<br>Stanford, California |
| 1 | Professor A. W. Straiton<br>University of Texas<br>Department of Electrical Engineering<br>Austin 12, Texas |
| 1 | Yale University<br>Department of Electrical Engineering<br>New Haven, Connecticut |
| 1 | Mr. James F. Trosch, Administrative Aide<br>Columbia Radiation Laboratory<br>Columbia University<br>538 West 120th Street<br>New York 27, New York |
| 1 | Dr. J.V.N. Granger<br>Stanford Research Institute<br>Stanford, California |
| 1 | Library<br>Central Radio Propagation Laboratory<br>National Bureau of Standards<br>Boulder, Colorado |

Technical Reports

1 Library of the College of Engineering
New York University
University Heights Library
University Heights 33, New York

1 Documents and Research Information Section
Raytheon Manufacturing Company
Engineering Equipment Division
148 California Street
Newton 58, Massachusetts

1 Professor Henry G. Bocker
School of Electrical Engineering
Cornell University
Ithaca, New York

1 M. A. Krivanich, Technical Advisor to
Deputy Chief
Ballistics Research Laboratory
White Sands Annex
White Sands P.G., New Mexico

1 Doris P. Baster
Head, Document Section
Technical Information Division
Naval Research Laboratory
Washington 25, D. C.

1 Dr. C. H. Papas
Department of Electrical Engineering
California Institute of Technology
Pasadena, California

1 Airborne Instrument Laboratory
Mineola
New York

1 Radiation Laboratory
Johns Hopkins University
1315 St. Paul Street
Baltimore 2, Maryland

1 Mr. Robert Turner
General Electric Company
Advanced Electronics Center
Cornell University
Ithaca, New York

Technical Reports

| | |
|---|---|
| 1 | Acquisitions Officer<br>ASTIA Reference Center<br>The Library of Congress<br>Washington 25, D. C. |
| 1 | Librarian<br>National Bureau of Standards Library<br>Connecticut Avenue and Van Ness Street, N. E.<br>Washington 25, D. C. |
| 1 | Secretary<br>Working Group on Semiconductor Devices, AGET<br>346 Broadway, 8th Floor<br>New York 13, N. Y. |
| 1 | Professor R. E. Norberg<br>Washington University<br>St. Louis, Missouri |